Simulating Stock Returns under switching regimes- a new test of market efficiency.*

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Abstract

A model of profits switches between four regimes with fixed probabilities; the rationally expected profits stream implies the stock market value. This efficient market model is not rejected by UK post-war time-series behaviour of either profits or the FTSE index.

Keywords: Regime switching, stock returns, efficient markets, rational expectations

JEL Classification: C15, C5, G14

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1 Introduction

Tests of market efficiency based on various forms of regression are beset with problems of interpretation — Minford and Peel (2002, ch. 14) — including the possibility of variable risk-premia, peso problems and rational bubbles.\(^1\) It is well known, for example, that issues of peso problems can make an important contribution to explaining apparent ex-post inefficiencies in asset markets. A notable example is that of Rietz (1988) who specifies the Mehra and Prescott (1985) model to include a low-probability, depression-like, third state in order to provide some explanation of the equity-premium puzzle. Given this type of result Markov switching models have become increasingly popular in analysis of asset prices. Markov models are able to generate a wide range of coefficients for skewness and kurtosis and serial correlation in mean and variance even when based on a very small number of underlying states (see e.g. Timmermann (2000)).\(^2\)

Of particular importance is the recent work by Guidolin and Timmermann (2005, 2005(a)). They study strategic asset allocation and consumption choice in the presence of regime switching in asset returns and find evidence that four separate regimes are required to capture the joint distribution of stock and bond returns.

It seems clear from previous analysis that switching models can mimic important properties displayed by stock markets. In this paper we build on that structure but complement it with a procedure inspired partly by the real business cycle (RBC) literature. Using quarterly data for the UK over the period 1963q2 to 2002q2 we create a regime switching model with four regimes (where the regimes represent high, normal and low growth, as well as a crash) with the probability of each regime constant over time, which generates a profits series. The rational expectation of the future profits is used to create the present discounted value which gives the implied stock market series, here the UK

\(^1\)Tests based on whether a rule could be found that would ‘beat the market’ also have problems of statistical assessment raised by Sullivan, Timmermann and White (2001)): how often, particularly with the aid of data-mining, might one not be able to find such a rule in any given set of data and even in a hold-out sample, especially again if the latter is subject, even subconsciously, to data-snooping.

\(^2\)See also e.g. Turner, Startz and Nelson (1989), Cecchetti, Lam and Mark (1990), Rydén, Teräsvirta, and Åsbrink (1998), Hamilton and Lin (1996),Whitelaw (2001) and Ang and Bekaert (2002).
FTSE. There is a constant probability of each future regime and the variance around future returns is fixed, as are any risk-premium terms. We have an efficient market world of rational agents by construction. As our profits regime is a latent process it cannot be observed. However we require that the profits series produced by this latent process must be consistent with the actual profits data for the sample period, in just the same way that the FTSE series produced by it must be consistent with the actual FTSE data for the same period. Thus our model has to pass a double test: that it can generate not merely the FTSE but also the profits processes.\textsuperscript{3} We use stochastic simulations to check on the model’s capacity to do so. At the end of the analysis we ask whether we can reject the hypothesis that the latent efficient market model is at work.

\section{The Working Hypothesis}

We can think of an economy (the UK in this case) where the capital stock generates the profits or dividends that are valued as equities (the FTSE). The profits are mainly driven by productivity shocks since variations in labour inputs mainly change wages while changes in the capital stock mainly dilute the equity base. So we shall assume that the fundamental, profits per share, can be identified with productivity (a more elaborate specification would involve a complete RBC model; but to minimise complexity we use this approximative assumption).

RBC models take the behaviour of productivity as exogenous, usually modelling it via some sort of univariate time-series. The empirical success of these models in matching the macroeconomic facts remains controversial. Their strength has been in supplying an explanation for certain salient macroeconomic correlations and cross-correlations; even

\textsuperscript{3}For a stock market to be efficient it must reflect the behaviour of the ‘fundamental’ efficiently. This fundamental must come from the actual data on profits or dividends which are being valued by the market. In other words the ‘driving process’ used in the models must be consistent with the actual processes at work in the data for efficiency to be established. It is not sufficient for there to be ‘some process’ if its implied behaviour is at variance with that of the data — if the implied moments of the process were different, then there would be inefficiency, with ‘excess’ or ‘inadequate’ variation compared with the fundamental.
though most macroeconomic modellers today rely on some source of substantial nominal rigidity to generate persistence in inflation and output, we would argue that RBC models remain capable of supplying a good account of macroeconomic growth and fluctuations (see Rebelo (2005)).

Here we suggest that the recognition of several regimes for productivity growth could be a helpful generalisation of the time-series process governing it. Thus for example one might identify periods of poor productivity growth — during wartime or poorly-adapted institutions (e.g. union power in the UK during the 1970s); periods of rapid productivity associated with surges of innovation (the industrial revolution, the computer revolution etc); and periods of normal growth, when innovation is being digested undisturbed by wars or dysfunctional institutions. Finally we included periods of ‘crash’ when profits drop off sharply and reduce the value of equities dramatically. The point has been made by Jorion and Goetzmann (1999) that from time to time around the world stock markets may suffer extreme loss or indeed total extinction because of a drastic interruption of profits from extreme negative events such as war or revolution. This possibility is present in even the most stable societies since such stability cannot ultimately be taken for granted.

Each of these productivity regimes we represent by an ARIMA (1,1,0) process with drift. The unit root represents the idea that productivity changes are in principle irreversible; the serial correlation the idea that once a change occurs it will be followed by further similar changes. The drift represents the mean growth of the regime. Each period the economy chooses one of the regimes with fixed probabilities — but the lagged effects of the previous regimes still are present. Thus if a war occurs, even when it is over its effects persist until they gradually disappear from the economy; meanwhile other shocks overlay them.\footnote{To calculate a reasonable order of magnitude for the probability of a crash occurring we used share price index data for 38 countries and calculated how many times the indices showed a quarterly growth rate of less than $-0.6$. Out of 4367 observations this occurred once, giving a probability of a crash equal to 0.023%. We think this is a small enough probability of a crash to be reasonable.}

Plainly we have no good theory of productivity growth and hence even RBC models for
which it is central treat it as exogenous. However, the discussion above seems a plausible improvement on the bald univariate processes generally assumed in RBC models. In our study we are unable to take our empirical work back beyond the actual profits series which we treat as the observable effect of productivity. Ideally one would like to identify the productivity series itself and integrate it within a full model of the economy; however such a more complete analysis lies beyond the scope of this study.

3 Our Test Procedure

We take the four regimes described above and assign the same iid normal-error to each. Each period the rational expectation of the future profits level is calculated and used to create the present discounted value (with a constant discount factor) which is the assumed FTSE. Notice that the conditional variance of profits around the future expectation is constant at all times since there is a constant probability of each future regime and hence of all future possible innovations; hence the variance surrounding future returns is equally fixed and with it any risk-premium terms attaching to FTSE valuation. Thus our set-up embodies all the standard assumptions of an efficient market world of rational agents.

At the next stage we search to find the best calibration of this model to the profits and FTSE data. Using the powerful method of grid search we choose the combination of parameters for the 4 ARIMA processes that minimises the distance between a linear combination of the moments of the simulated FTSE and the actual FTSE and those of the simulated profits process from those of the actual process.\(^5\)

Using the best set of parameters the composite process is then simulated stochastically in 50000 runs of 150 periods. With these stochastic simulations we then carry out the tests described at the start of this paper. We wish to know whether the profits and FTSE data can each be regarded as a sample drawing from this model. We look first at the profits and FTSE sample moments; do they lie outside the 95% confidence limits of the null hypothesis distributions (found from the 50000 generated samples)? Secondly, we

\(^5\)The method we use here is similar to Simulated Method of Moments, though we do not vary the weights on the moments in the critical value.
look at the profits ARIMA process and the FTSE GARCH process describing respectively
the profits and the FTSE’s dynamics; do the parameters of these processes lie outside
the 95% confidence limits obtained from the 50000 generated samples?

3.1 Model details

The four regimes are each assumed to have their own mean \((c_1, c_2, c_3, c_4)\) and equal
standard deviation \((\sigma_\eta)\). The low growth, high growth and crash regimes have the prob-
ability of occurrence \(\pi_1, \pi_3, \pi_4\). For each period we choose the regime at random and the
corresponding growth \((c_i)\), and then choose a random number \(\eta \sim N(0, \sigma_\eta)\). This is then
used to calculate our generated FTSE as follows. Starting with initial values of profits
\(w_1 = w_2 = 1\) each period of our FTSE series is given by

\[
A_t = \exp(w_t - 1) + \sum_{i=1}^{150} \beta^i \exp(E_t w_{t+i} - 1) \quad \text{for } t = 3, 152 \tag{1}
\]

where

\[
\begin{align*}
w_t &= w_{t-1} + \rho_i \Delta w_{t-1} + \eta_t + c_i \\
E_t w_{t+1} &= w_t + \rho_i \Delta w_t + \sum_{i=1}^{4} \pi_i c_i \\
E_t w_{t+2} &= E_t w_{t+1} + \rho_i (E_t w_{t+1} - w_t) + \sum_{i=1}^{4} \pi_i c_i \\
E_t w_{t+3} &= E_t w_{t+2} + \rho_i (E_t w_{t+2} - E_t w_{t+1}) + \sum_{i=1}^{4} \pi_i c_i \\
&\vdots
\end{align*}
\]

and \(\beta = 0.99\).

Repeating this for the length of the sample gives us a complete ‘pseudo-sample’ se-
ries for both profits and the FTSE; this operation is repeated 50000 times to generate
this number of pseudo-samples. We want to find which combination of parameters gives
the best match to the actual FTSE and profits series; since these series are both in-
tegrated of order 1 we examine them in log-differenced form, \(DFT = \Delta \log FTSE_t\) and
DPROF=Δ\log(PROF_t). Our grid search is guided by the difference between a weighted average of the log-differenced series moments in the data and in a subset of our pseudo-samples (we use a subset of 50). We search for the parameters which minimise this difference.

For our test procedure we estimate on the actual data the first four moments and also the best available parsimonious time-series descriptions of the two data series, DFT and DPROF: for the first an ARCH-ARMA(1,0), and for the second an ARMA(1,0). Thus:

\[
\Delta \log FTSE_t = \beta_1 \Delta \log FTSE_{t-1} + \epsilon_t \tag{2}
\]

with a ARCH(1) representation of the residuals of this regression\(^6\), and

\[
\Delta \ln(PROF_t) = \alpha_1 + \alpha_2 \Delta \ln(PROF_{t-1}) + \epsilon^1_t \tag{3}
\]

The estimated coefficients from this regression are then used as the normal growth rate (regime 2) and the serial correlation (\(\rho_{all}\)). We then estimate the moments and the corresponding equations on the generated two series, to obtain the 95\% confidence limits for both moments and equation parameters. The working hypothesis is rejected if any of these lie outside the 95\% interval.

4 Results

We investigated this model initially in a variety of versions with less than four regimes. However, they all in various ways failed to match jointly the properties of the FTSE and the profits data. Turning to the model of four regimes described above, we found that the combination of parameters from the search algorithm that minimised the critical value were

\(^6\)We initially estimated \(\Delta \log FTSE_t = \beta_1 + \beta_2 \Delta \log FTSE_{t-1} + \epsilon_t\) with a GARCH(1,1) representation, but found that \(\beta_1\) and the GARCH parameter were not statistically significant.
\[
\begin{align*}
\rho_1 &= -0.2 & \rho_2 &= -0.10868384 & \rho_3 &= 0.3 & \rho_{\text{crash}} &= 0.9 \\
\c_1 &= -0.063978 & \c_2 &= 0.006414 & \c_3 &= 0.1 & \c_4 &= -0.3 \\
\pi_1 &= 0.07828 & \pi_2 &= 0.91172 & \pi_3 &= 0.01 & \pi_4 &= 0.0002 \\
\sigma_\eta &= 0.003267
\end{align*}
\]

Note that here for the crash regime alone the \( \rho \) parameter is set at \( \rho_{\text{crash}} = 0.9 \), an important element in the process of matching the moments. We had to raise the \( \rho \) parameter for a crash because the variance of the FTSE is large compared to the variance of profits, so that a larger \( \rho \) was required to match the variance of the FTSE. Also, the \( \rho \)'s for the other regimes are calculated so that the average \( \rho \) is equal to the estimated value. The results for this model are in Table 1.

As one can see from Table 1, the model with regime switching across four regimes including a crash matches all the properties of both the FTSE and the profits series, in the sense that at a 95% confidence interval it cannot be rejected by our chosen descriptive measures of these two data series. Figure 1 shows the actual FTSE series along with a random selection of our generated FTSE series. Figure 1 shows that this model produces series that have the peaks and troughs associated with the actual FTSE series. Figure 2 show the histograms of the parameters of our estimated FTSE and Profits series. The red line is the actual value, and the black lines the 2.5% limits.

5 Conclusions

We generated a profits series with a regime switching model with four regimes (where the regimes represent high, normal and low growth, as well as a crash) with the probability of each regime constant over time. The rational expectation of the generated future profits was used to create the present discounted value which gives the implied stock market series, say, FTSE. We found that this model was able to generate a series that matched the properties of the actual FTSE and actual profits series, in the sense that we cannot reject at the 95% level of confidence the proposition that both the profits and the FTSE samples were generated by our model.
We have shown here that the hypothesis of efficiency, if constructed to incorporate the possibility of extreme events, can mimic the behaviour of the FTSE. It is of course possible that alternative hypotheses, such as behavioural finance, can also explain these facts; but that issue and whether such explanations can be considered better ones, is an issue that must be deferred to further research.
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Table 1: Results for the markov switching model with a crash
Figure 1: Actual FTSE and a Random Selection of Generated FTSE
Figure 2: Histograms of Generated FTSE and Profits Parameters
References


