# 1. Simple Classical and Keynesian Models

## Tutorial 1

Consider a closed economy characterised by the following equations:

$$y^d = c + i + g,$$

$$c = a + b(y - t), \qquad a > 0, 0 < b < 1$$

$$i = i_0 - hr, \qquad i_0, h > 0$$

$$y^d = y,$$

$$\frac{M^d}{P_0} = m_0 + ky - lr, \qquad k, l > 0$$

$$M^d = M$$

Symbols have their usual meaning.

Given $a = 100,\ b = 0.8,\ t = 400,\ i_0 = 500,\ h = 50,\ g = 400,$
$m_0 = 500,\ k = 0.2,\ l = 25, M = 520,\ P_0 = 1.$

1. Calculate equilibrium income and equilibrium interest rate (a) in terms of algebraic symbols (b) numerically.
2. Calculate equilibrium interest rate when there is a simultaneous rise in

(i) government expenditure by 200 units and money supply by 125 units,
(ii) government expenditure by 200 units and money supply by 225 units.

Comment on (i) & (ii) in terms of IS & LM curves.

3. What happens to equilibrium income and interest rate when a rise in government spending by 200 units is financed (i) by lumpsum taxes worth 50 units and the rest by money creation, (ii) equally by lumpsum taxes and money creation.

# Solution

1) Substituting for $c$ & $i$ in $y^d$ yields:

$$y = a + b(y - t) + i_0 - hr + g$$
$$(1 - b)y = a - bt + i_0 - hr + g$$

or

$$y = \frac{1}{1-b}[a - bt + i_0 - hr + g] \qquad \text{(IS curve)} \qquad 1.1$$

Similarly from the last two equations we get:

$$\frac{M}{P_0} = m_0 + ky - lr$$

or

$$y = \frac{1}{k}\left[\frac{M}{P_0} - m_0 + lr\right] \qquad \text{(LM curve)} \qquad 1.2$$

Equating (1.1) and (1.2) gives equilibrium real interest rate:

$$\frac{1}{1-b}[a - bt + i_0 - hr + g] = \frac{1}{k}\left[\frac{M}{P_0} - m_0 + lr\right]$$

$$\Rightarrow \left[\frac{1}{k}l + \frac{1}{1-b}h\right]r = \frac{1}{1-b}[a - bt + i_0 + g] - \frac{1}{k}\left[\frac{M}{P_0} - m_0\right]$$
$$\Rightarrow [(1-b)l + kh]r = k[a - bt + i_0 + g] - (1-b)\left[\frac{M}{P_0} - m_0\right]$$
$$r = \frac{k[a - bt + i_0 + g]}{(1-b)l + kh} - \frac{(1-b)(\frac{M}{P_0} - m_0)}{(1-b)l + kh}$$

$$r^* = \frac{1}{(1-b) + \frac{kh}{l}}\left[\frac{k}{l}(a - bt + i_0 + g)\right] - \frac{1}{(1-b) + \frac{kh}{l}}\left[\frac{1-b}{l}\left(\frac{M}{P_0} - m_0\right)\right] \qquad 1.3$$

Substituting (1.3) in (1.2) yields equilibrium real income:

$$y = \frac{1}{k}\left[\frac{M}{P_0} - m_0\right] + \frac{1}{(1-b) + \frac{kh}{l}}\left[(a - bt + i_0 + g) - \frac{1-b}{k}\left(\frac{M}{P_0} - m_0\right)\right]$$

or

$$y^* = \left( \frac{1}{(1-b) + \frac{kh}{l}} \right) \left\{ (a - bt + i_0 + g) + \frac{h}{l} \left( \frac{M}{P_0} - m_0 \right) \right\} \qquad 1.4$$

With $a = 100,\ b = 0.8,\ t = 400,\ i_0 = 500,\ h = 50,\ g = 400,\ m_0 = 500,\ k = 0.2,$ $l = 25, M = 520,\ P_0 = 1.$

Substituting in (1.3) & (1.4) respectively yields:

$$r^* = 8.8\%$$

and

$$y^* = 1200$$

2(i) With $\partial g = 200,\ \partial M = 125$ we have:

$$\partial y^* = 750 \text{ or } y^* = 1950$$
$$\partial r^* = 1\% \text{ or } r^* = 9.8\%$$

2(ii) With $\partial g = 200,\ \partial M = 225$ we have:

$$\partial y^* = 1083.33 \text{ or } y^* = 2283.33$$
$$\partial r^* = -0.33\% \text{ or } r^* = 8.47\%$$

3(i) With $\partial g = 200,\ \partial M = 150,\ \partial t = 50$ we have:

$$\partial y^* = 766.67 \text{ or } y^* = 1966.67$$
$$\partial r^* = 0.13\% \text{ or } r^* = 8.93\%$$

3(ii) With $\partial g = 200,\ \partial M = 100,\ \partial t = 100$ we have:

$$\partial y^* = 533.33 \text{ or } y^* = 1733.33$$
$$\partial r^* = 0.27\% \text{ or } r^* = 9.07\%$$

# Tutorial 2

Consider an economy characterised by the following equations, where symbols have their usual meaning. Equation 1 describes the production function, equation 2 gives labour demand, equation 3 gives labour supply. Equations 4,5, and 6 denote the consumption function, the investment function and the money demand function respectively. Aggregate demand is the sum of expenditure on consumption, investment and government expenditure. The labour market clears through wage-price flexibility. (The commodity and money markets also clear). Assume government expenditure to be fixed at 7, the money supply at 50, and that the initial capital stock is 64. The tax rate is 0.25.

$$y = 2\sqrt{nk} \qquad 2.1$$

$$n^d = 19 - 1.5\left(\frac{W}{P}\right) \qquad 2.2$$

$$n^s = -2 + 9\left(\frac{W}{P}\right) \qquad 2.3$$

$$c = 11.6 + 0.8(1-t)y \qquad 2.4$$

$$i = 10 - 20r \qquad 2.5$$

$$\frac{M}{P} = 15.1 + 0.1y - 10r \qquad 2.6$$

1. Solve for (i) equilibrium employment, (ii) full-employment output, (iii) the price level and the nominal wage.

2. What happens to (i), (ii) and (iii) above, when

(a) government expenditure is raised to 9?

(b) money supply is raised to 57?

3. Give economic explanations for your results in 1 & 2 above.

## Solution

1) In order to solve for equilibrium employment equate $n^d$ and $n^s$ i.e., labour market equilibrium condition should give us equilibrium real wage $(\frac{W}{P})^*$ and equilibrium employment $(n^*)$.

$$\Rightarrow \; 19 - 1.5\left(\frac{W}{P}\right) = -2 + 9\left(\frac{W}{P}\right)$$

$$\Rightarrow 21 = 10.5\left(\frac{W}{P}\right)$$

$$\left(\frac{W}{P}\right)^* = 2 \qquad \text{(equilibrium real wage)} \qquad 2.7$$

Substituting (2.7) in the equation for $n^s$ gives equilibrium employment:

$$\Rightarrow n^S = -2 + 9\left(\frac{W}{P}\right)$$

$$n^* = 16 \qquad 2.8$$

Since capital stock is given, substituting $k$ and $n^s$ in (2.1) yields full-employment output:

$$\Rightarrow \; y = 2\sqrt{nk}$$

or

$$y^* = 64 \qquad 2.9$$

Since aggregate demand is the sum of expenditure on consumption, investment and government expenditure i.e.,

$$y = c + i + g$$

$$\Rightarrow 64 = 11.6 + 0.8(1 - 0.25)64 + 10 - 20r + 7$$

or

$$-3 = -20r$$

$$r = 0.15\% \qquad \text{(equilibrium interest rate)} \qquad 2.10$$

In order to find out the price level substitute $m = 50,\ r = 0.15\%,$ and $y = 64$ in equation (2.6):

$$\Rightarrow \frac{50}{P} = 15.1 + 0.1(64) - 10(0.15)$$

$$\Rightarrow 50 = 20P$$

$$P = 2.5 \qquad \text{(the price level)} \qquad 2.11$$

Substituting for $P$ in the expression for real wage yields nominal wage $W$:

$$\Rightarrow \frac{W}{2.5} = 2$$

$$W = 5 \qquad \text{(nominal wage)} \qquad 2.12$$

2(a) When government expenditure is raised to 9;

$$y = c + i + g$$

$$\Rightarrow 64 = 11.6 + 0.8(1 - 0.25)64 + 10 - 20r + 9$$
$$\Rightarrow -5 = -20r$$

$$r = 0.25\% \qquad 2.13$$

Substituting for $r$ in equation 2.6 yields solution for the price level:

$$\Rightarrow \frac{50}{P} = 15.1 + 0.1(64) - 10(0.25)$$

$$P = 2.63 \qquad 2.14$$

Given that $P = 2.63$, substituting in the real wage equation yields solution for nominal wage:

$$\Rightarrow \frac{W}{2.63} = 2$$

$$W = 5.26 \qquad 2.15$$

2(b) When money supply is raised to 57;

$$\Rightarrow \frac{57}{P} = 15.1 + 0.1(64) - 10(0.15)$$

$$P = 2.85 \qquad 2.16$$

Given that $P = 2.85$, substituting in the real wage equation yields solution for nominal wage:

$$\Rightarrow \frac{W}{2.85} = 2$$

$$W = 5.7 \qquad 2.17$$

3) This exercise illustrates the issue of 'classical dichotomy'. The real variables are determined for equations (2.1), (2.2) and (2.3) which solve for $n^*$, $(\frac{W}{P})^*$, and $y^*$. Once equilibrium output is obtained, the goods market equilibrium condition determines equilibrium interest rate and the money market equilibrium determines the general price level. With a rise in government expenditure the IS curve shifts upwards while the LM curve shifts downwards leading to a higher interest rate and price level with no change in the real variables. On the otherhand a rise in money supply initially shifts the LM curve upwards and then shifts back to its original position, so that the only change is on the general price level which is now higher.

# Tutorial 3

1. Take the model of tutorial sheet 1, in terms of algebra derive the aggregate demand curve for the economy. What is the value of aggregate demand corresponding to a price level of 2?

2. Consider an equilibrium model of a closed economy characterised by the following equations:

$$m - p = ay - br \qquad 3.1$$

$$y = c - dr + eg - ft \qquad 3.2$$

$$y^s = h + j(p - p^e) \qquad 3.3$$

where

$a = 0.75$ $\quad b = 20$ $\quad c = 80$

$d = 20$ $\quad e = 5$ $\quad f = 2$

$h = 150$ $\quad j = 4$

Exogenous variables:

$$m = 80 + \varepsilon$$
$$g = 40 + u$$
$$t = 20 + v$$

All variables above (except for the interest rate) are in natural logs. The superscript $e$ refers to an expected value*. The expected values of ε, $u$ and $v$ are zero footnote .

1. What is the equation for the aggregate demand curve?

2. What is the equation for the aggregate supply curve?

3. What is the expected price level?

4. What is the expected interest rate?

## Solution

1) Equilibrium real income in tutorial 1 was given by:

$$y^* = \left( \frac{1}{(1-b) + \frac{kh}{l}} \right) \left\{ (a - bt + i_0 + g) + \frac{h}{l} \left( \frac{M}{P_0} - m_0 \right) \right\}$$

$$\Rightarrow y^* = \left( \frac{1}{0.2 + \frac{10}{25}} \right) \{ (100 - 320 + 500 + 400) + [2(260 - 500)] \}$$

$$y^* = 333.33$$

Equilibrium real interest rate in tutorial 1 was given by:

$$r^* = \frac{1}{(1-b) + \frac{kh}{l}} \left[ \frac{k}{l} (a - bt + i_0 + g) \right] - \frac{1}{(1-b) + \frac{kh}{l}} \left[ \frac{1-b}{l} \left( \frac{M}{P_0} - m_0 \right) \right]$$

$$\Rightarrow r^* = \frac{1}{0.6} \left[ \frac{0.2}{25}(680) - \frac{0.2}{25}(-240) \right]$$

$$r^* = 12.27\%$$

2) IS-LM intersection should give us aggregate demand schedule. From equation (3.1):

$$y = \frac{1}{a}[(m - p) + br] \qquad \text{(LM curve)} \qquad 3.4$$

We know from equation (3.2);

$$y = c - dr + eg - ft \qquad \text{(IS curve)} \qquad 3.5$$

Equating (3.1) and (3.2) yields equilibrium real interest rate:

$$\frac{1}{a}[(m-p)+br] = c - dr + eg - ft$$

$$\Rightarrow r\left(\frac{b}{a}+d\right) = c + eg - ft - \left(\frac{m-p}{a}\right)$$

$$r^* = \left(\frac{a}{b+ad}\right)\left[c + eg - ft - \left(\frac{m-p}{a}\right)\right] \qquad 3.6$$

Substituting (3.6) in the LM schedule yields:

$$\Rightarrow y = \frac{1}{a}(m-p) + \frac{b}{a}\left[\left(\frac{a}{b+ad}\right)\left(c + eg - ft - \left(\frac{m-p}{a}\right)\right)\right]$$

Collecting terms in $(m-p)$ yields:

$$y = (m-p)\left(\frac{d}{b+ad}\right) + \frac{b}{b+ad}(c + eg - ft)$$

$$y^{AD} = \left(\frac{1}{b+ad}\right)[d(m-p) + b(c + eg - ft)] \qquad 3.7$$

Note that the aggregate supply curve is already given. Equation (3.3) is the Phillips curve or the aggregate supply curve.

$$y^{AS} = h + j(p - p^e) \qquad 3.8$$

If we equate expected aggregate demand with expected aggregate supply we get expected price level. If we run the expectations operator along the aggregate supply curve we get:

$$y^{AS^e} = h \equiv y^* \qquad \text{(potential or full-employment output)} \qquad 3.9$$

Similarly running the expectations operator along the aggregate demand curve and equating the resulting expression with expected aggregate supply yields expected price level:

$$p^e = m^e + \frac{\beta}{\alpha}(c + eg^e - ft^e) - \frac{y^*}{\alpha} \qquad 3.10$$

where $\alpha = \frac{d}{b+ad}$ and $\beta = \frac{b}{b+ad}$

In order to compute expected interest rate just run the expectations operator along the interest rate equation that we have already derived i.e.,

**Definition** *If c is any constant, it follows directly that* $E[cg] = cE[g]$.

$$r^{*e} = \left(\frac{a}{b+ad}\right)\left[c + eg^e - ft^e - \left(\frac{m^e - p^e}{a}\right)\right] \qquad 3.11$$

# Tutorial 4

1. Consider a situation where the inflation rate ($\pi$) in year 0 is 0%, which coincides with what people expected ($\pi^e$) at the beginning of that year. Now let inflation start rising by 1% each year, from year 1 onwards. Let agents form expectations about inflation using the following rule:

$$\pi_t^e - \pi_{t-1}^e = 0.5(\pi_{t-1} - \pi_{t-1}^e)$$

(i) Find the values for expected inflation in years 1, 2, 3 and 4.

(ii) Comment on the forecasting rule given that actual inflation rises every year.

(iii) Can you infer from the above what would happen if actual inflation *fell* by 1% every year?

2. Consider the following model of a closed economy where all variables (except for the interest rate) are in natural logs. Symbols have usual meanings. The superscript e refers to an expected value.

$$\overline{m}_t - p_t = 0.5y_t - 20R_t \quad 4.1$$

$$y_t = c_0 - 25r_t + (g - \tau)_t \quad 4.2$$

$$\pi_t = 0.1(y_t - y^*) + \pi_t^e \quad 4.3$$

$$R_t = r_t + \pi_{t+1}^e \quad 4.4$$

$$\pi_t \equiv p_t - p_{t-1} \quad 4.5$$

$\overline{m}_t = 200,\ \overline{m}_{t-1} = 180,\ c_0 = 400,\ g - \tau = 200,\ y^* = 500.$

Assume full equilibrium with all markets clearing, where inflation expectations are realised and the inflation rate is equal to the growth in the money supply.

a) Solve for
(i) the equilibrium inflation rate,
(ii) the equilibrium nominal interest rate,
(iii) the equilibrium price level.

b) What would be the long-run effect of increasing the budget deficit to 300?

# Solution

1(i) Consider the adaptive expectations rule:

$$\pi_t^e - \pi_{t-1}^e = 0.5(\pi_{t-1} - \pi_{t-1}^e)$$

Let (t-1) be year 0. Then $\pi_0 = 0$ and $\pi_0^e = 0$ as well.

$$\Rightarrow \pi_1^e - \pi_0^e = 0.5(\pi_0 - \pi_0^e)$$

$$\pi_1^e = 0$$

According to the forecasting rule:

$$\pi_{t+1}^e - \pi_t^e = 0.5(\pi_t - \pi_t^e)$$

$$\Rightarrow \pi_2^e - \pi_1^e = 0.5(\pi_1 - \pi_1^e)$$

Given that $\pi_1 = 1$ and having found $\pi_1^e = 0$ above, we have

$$\pi_2^e = 0.5$$

Similarly for year 3 we have;

$$\Rightarrow \pi_3^e - \pi_2^e = 0.5(\pi_2 - \pi_2^e)$$

Given that $\pi_2 = 2$ and having found $\pi_2^e = 0.5$ above, we have

$$\pi_3^e = 1.25$$

For year 3 we get;

$$\Rightarrow \pi_4^e - \pi_3^e = 0.5(\pi_3 - \pi_3^e)$$

Given that $\pi_3 = 3$ and having found $\pi_3^e = 1.25$ above, we have

$$\pi_4^e = 2.125$$

| Year | $\pi$ | $\pi^e$ |
|---|---|---|
| 0 | 0 | 0 |
| 1 | 1 | 0 |
| 2 | 2 | 0.5 |
| 3 | 3 | 1.25 |
| 4 | 4 | 2.125 |

1(ii) Actual inflation rises by 1% every year, but because of the forecasting rule, there is *systematic underprediction* of the inflation rate. In otherwords agents make systematic forecasting errors.

1(iii) If actual inflation fell by 1% every year there would have been a *systematic overprediction* of the inflation rate.

2(a) Note that $\pi_t = \frac{P_t - P_{t-1}}{P_{t-1}} \simeq \ln P_t - \ln P_{t-1}$

Similarly $\frac{m_t - m_{t-1}}{m_{t-1}} \simeq \ln m_t - \ln m_{t-1}$

Since $\overline{m}_t - \overline{m}_{t-1} = 20 \ (= \pi_t, \text{ given})$

$$\Rightarrow \pi_t = \pi_t^e = 20 \text{ and } y^* = 500$$

$$y_t = 500 \quad \text{(from equation 4.3)}$$

From equation (4.2) we can determine the real interest rate:

$$\Rightarrow 500 = 400 - 25r_t + 200$$

$$r_t = 4$$

Since all markets clear and inflation expectations are realised:

$$\pi_{t+1}^e = \pi_t^e = \pi_t = 20$$

It follows that $R_t = 24$ from equation (4.4).

From equation (4.1) substituting for $\overline{m}_t$, $y_t$ and $R_t$ yields:

$$\Rightarrow 200 - p_t = 0.5(500) - 20(24)$$

$$p_t = 430$$

2(b) In order to understand the long-run effect of an increase in budget deficit substitute $g - \tau = 300$ in equation (4.2).

$$\Rightarrow 500 = 400 - 25r_t + 300$$

$$R_t = 28$$

So, in the long-run where $\pi_t^e = \pi_t$, $y_t = y^*$, and $\pi_t = \overline{m}_t - \overline{m}_{t-1}$, a higher budget deficit has no effect on output. It simply crowds out private investment through a rise in the real interest rate. A rise in $r_t$ implies a rise in $R_t$ so that the general price level $(p_t)$ would have to rise. From equation (4.1):

$$\Rightarrow\ 200 - p_t = 0.5(500) - 20(28)$$

$$p_t = 510$$

The price level has to rise to bring the money market back to equilibrium. Hence, there is a shift of the IS curve (upwards) and also of the LM curve (downwards).

# Tutorial 5

1. Consider an equilibrium model of a closed economy (with government purchases and income tax assumed exogenous) characterised by the following equations:

$$Y = C + \overline{I} + \overline{G} \qquad \text{(GNP identity)}$$
$$C = \alpha + \beta(1 - t)Y \qquad \text{(Consumption function)}$$

i) Compute the government expenditure multiplier?

2. Consider an equilibrium model of a closed economy characterised by the following equations:

$$Y = C + I + \overline{G} \qquad \text{(GNP identity)}$$
$$C = \alpha + \beta(1 - t)Y \qquad \text{(Consumption function)}$$

$$I = d - ei \qquad \text{(Investment function)}$$
$$M_s = f - hi \qquad \text{(Speculative demand for money)}$$
$$M = M_s + M_t \qquad \text{(Total demand for money)}$$
$$M_t = kY \qquad \text{(Cambridge cash balance equation)}$$

Given $M$, $\overline{G}$, $t$ and the parameters $\alpha$, $\beta$, $d$, $e$, $h$, and $k$, the system determines the values of $Y$, $C$, $I$, $i$, $M_s$, and $M_t$.

a) Derive an expression for output?

b) Compute the government expenditure multiplier?

c) What happens to output when income velocity of circulation (average number of times money changes hands during a given year) falls?

d) Derive an expression for "the money multiplier"?

## Solution

1(i) In order to compute the government expenditure multiplier substitute the consumption function into the GNP identity (with investment and government expenditure being exogenous):

$$\Rightarrow\ Y = \alpha + \beta(1-t)Y + \overline{I} + \overline{G}$$

$$Y = \frac{1}{1-\beta(1-t)}(\alpha + \overline{I} + \overline{G})$$

$$\frac{\partial Y}{\partial \overline{G}} = \frac{1}{1-\beta(1-t)} \qquad \text{(government expenditure multiplier)}$$

2(a) Substitute the consumption and investment function into the GNP identity to get:

$$\Rightarrow\ Y = \alpha + \beta(1-t)Y + d - ei + \overline{G}$$

$$Y = \frac{1}{(1-\beta(1-t))}(\alpha + d - ei + \overline{G}) \qquad 5.1$$

Substituting for money demand yields:

$$\Rightarrow\ M = f - hi + kY$$

$$i = \frac{1}{h}(f + kY - M) \qquad 5.2$$

Substituting equation (5.2) in (5.1) yields:

$$\Rightarrow\ Y = \frac{1}{(1-\beta(1-t))}\left(\alpha + d - \frac{e}{h}(f + kY - M) + \overline{G}\right)$$

$$\Rightarrow\ Y\left(1 + \frac{\frac{e}{h}k}{(1-\beta(1-t))}\right) = \frac{1}{(1-\beta(1-t))} \times \left(\alpha + d - \frac{e}{h}(f - M) + \overline{G}\right)$$

$$Y = \frac{1}{(1-\beta(1-t)) + \frac{ek}{h}}\left(\alpha + d - \frac{ef}{h} + \frac{eM}{h} + \overline{G}\right) \qquad 5.3$$

b) The government expenditure multiplier is given by:

$$\frac{\partial Y}{\partial \overline{G}} = \frac{1}{(1-\beta(1-t)) + \frac{ek}{h}}$$

c) Note that the last equation of the model is nothing but the Cambridge cash balance equation:

$$M_t = kY$$

$$\Rightarrow k = \frac{M_t}{Y} = \frac{1}{V} = \text{inverse of the income velocity of circulation}$$

So, lower '*V*' means higher '*k*'. A large *k* reduces the multiplier because it makes for a heavy transaction drain as income rises, leading to a fall in money supply, a rise in the interest rate, and hence "crowding out" private investment.

d) In order to compute the money multiplier differentiate equation (5.3) w.r.t money stock to get;

$$\frac{\partial Y}{\partial M} = \frac{\frac{e}{h}}{(1-\beta(1-t)) + \frac{ek}{h}}$$