6. Open Economy Models

Tutorial 45

1) Consider an open-economy model (with government purchases and income tax assumed to be exogenous) and fixed exchange rates;

$$Y = C + \overline{I} + \overline{G} + (E - M)$$
$$C = \alpha + \beta(1 - t)Y$$
$$M = mC$$

where E stands for exports, M for imports, m for expenditure on imported consumer goods as a fraction of consumption expenditure, and β denotes the marginal propensity to consume.

i) Compute the government expenditure multiplier and comment?

1(i) Substitute the consumption function and the import equation into the GNP identity to get:

$$\Rightarrow Y = C + \overline{I} + \overline{G} + (E - mC)$$
$$\Rightarrow Y = (1 - m)(\alpha + \beta(1 - t)Y) + \overline{I} + \overline{G} + E$$

$$Y = \frac{1}{(1 - \beta(1 - t)(1 - m))}(\alpha(1 - m) + \overline{I} + \overline{G} + E)$$

Comparing the open-economy multiplier with that of the closed economy shows that the multiplier has been reduced by import leakage. Thus in an open-economy, the effectiveness of fiscal policy is reduced substantially by the existence of trade leakages.

Tutorial 46

Consider a small open-economy characterised by the following equations.

Symbols have their usual meaning.

$y_t = -\alpha r_t - \delta e_t + k\theta_t$	IS curve	46.1
$\Delta\theta_t = -qe_t - \mu y_t$	Wealth definition	46.2
$y_t = \sigma e_t + p\pi_t^{ve}$	Phillips curve	46.3
$r_t = r_{f,t} - E_t e_{t+1} + e_t$	UIP condition	46.4

1. Solve the following model of a fixed exchange rate for e_t, the real exchange rate. Represent π_t^{ve} as ϵ_t; find the Muth solution for $e_t = \sum_{i=0}^{\infty} q_i \varepsilon_{t-i}$. $\left(\text{Set } r_{f,t} = 0\right)$. Interpret your solution (under fixed exchange rates $\pi_t = e_t - \overline{S_t} + \overline{\pi}_{f,t}$). Assume $\overline{S_t} = \overline{\pi}_{f,t} = 0$.

Solution

Substituting (46.4) into (46.1) for r_t yields:

$$y_t = -\alpha\left(-E_t e_{t+1} + e_t\right) - \delta e_t + k\theta_t$$

or

$$y_t = \alpha E_t e_{t+1} - e_t(\alpha + \delta) + k\theta_t \qquad 46.5$$

Substituting (46.5) in (46.3) for y_t yields:

$$\sigma e_t + p\epsilon_t = \alpha E_t e_{t+1} - e_t(\alpha + \delta) + k\theta_t \qquad 46.6$$

Rewrite (46.2) as

$$k\theta_t - k\theta_{t-1} = -kqe_t - k\mu y_t \qquad 46.7$$

Lag (46.6) one period and substitute in (46.7) for $k\theta_{t-1}$ and $k\theta_t$ to get

$$e_t(\alpha + \delta + \sigma) + p\epsilon_t - \alpha E_t e_{t+1} - e_{t-1}(\alpha + \delta + \sigma) - p\epsilon_{t-1} + \alpha E_{t-1} e_t$$
$$= -kqe_t - k\mu(\sigma e_t + p\epsilon_t)$$

or

$$e_t(\alpha+\delta+\sigma+kq+k\mu\sigma)-(\alpha+\delta+\sigma)e_{t-1}-\alpha E_t e_{t+1}+\alpha E_{t-1}e_t$$
$$= -p(1+k\mu)\epsilon_t+p\epsilon_{t-1} \qquad 46.8$$

Muth solution for $e_t = \sum_{i=0}^{\infty} q_i \varepsilon_{t-i}$.
Collecting terms in:

$$(\epsilon_t) \;:\; \sigma' q_0-\alpha q_1 = -p(1+k\mu) \qquad \text{A}$$
$$\text{where } \sigma' = (\alpha+\delta+\sigma+kq+k\mu\sigma)$$
$$(\epsilon_{t-1}) \;:\; \sigma' q_1-(\alpha+\delta+\sigma)q_0-\alpha q_2+\alpha q_1 = p \qquad \text{B}$$
$$\left(\epsilon_{t-i},\, i\geq 2\right) \;:\; q_{t+i}-\left(2+\frac{\delta+\sigma+kq+k\mu\sigma}{\alpha}\right)q_i+\left(\frac{\delta+\sigma+\alpha}{\alpha}\right)q_{i-1} = 0$$

The solution for the difference equation:

$$q_{t+i} = A_1\lambda_1^i+A_2\lambda_2^i \qquad \left(\text{for } i\geq 2\right)$$
$$q_2 = A_1\lambda_1+A_2\lambda_2$$
$$q_1 = A_1+A_2$$

The roots of the quadratic equation are

$$\lambda_1,\,\lambda_2 = \left(1+\frac{1}{2}x\right)\pm\frac{1}{2}\sqrt{x^2+4\left(\frac{kq+k\mu\sigma}{\alpha}\right)}$$
$$\text{where } x = \frac{\delta+\sigma+kq+k\mu\sigma}{\alpha}$$
$$\text{i.e., } \lambda_1 = 1+\frac{1}{2}x+\frac{1}{2}(x+?) > 1$$
$$\lambda_2 = 1+\frac{1}{2}x-\frac{1}{2}(x+?) < 1$$

Setting terminal condition gives $A_1 = 0$. $\therefore\; q_2 = q_1\lambda_2$.
Solving (A) and (B) for q_0 and $q_1$yields:

$$q_0 = \frac{-p((1+k\mu)(\delta+\sigma+kq+k\mu\sigma)+\alpha k\mu)}{\sigma^2+\delta^2+k^2(q\mu+\sigma)^2+\sigma\alpha+2\sigma\delta+2(\delta+\sigma+\alpha)k(q+\mu\sigma)+\sigma'}$$
$$q_1 = \frac{-pk(\mu(\alpha+\delta)-q)}{\sigma^2+\delta^2+k^2(q\mu+\sigma)^2+\sigma\alpha+2\sigma\delta+2(\delta+\sigma+\alpha)k(q+\mu\sigma)+\sigma'}$$

$q_0 < 0$ and $q_1 < 0$ if $q < \mu(\alpha+\delta)$ i.e., if the slope of ISBB curve is flatter than the slope of XM in the text.

Solution for e_t is therefore

$$e_t = q_0 \epsilon_t + \frac{q_1 \epsilon_{t-1}}{1 - \lambda_2 L}$$

or

$$e_t = q_0 \epsilon_t + (q_1 - \lambda_2 q_0) \epsilon_{t-1} + \lambda_2 e_{t-1}$$

Tutorial 47

Extension to the Open Economy:

Consider a modified version of Backus et al. (1994) real business cycle (RBC) model of the open economy. Assume that the representative agent in each country (home and foreign) are characterised by a utility function of the form:

$$U = MaxE_t\left[\sum_{t=0}^{\infty}\beta^t(\theta C_t^{-\rho} + (1-\theta)(L_t)^{-\rho})^{-\frac{1}{\rho}}\right]$$

where notations have their usual meaning.

With respect to technology each country specialises in the production of a single good, labelled *a* for country 1 (UK) and *b* for country 2 (rest of the world). Each good is produced using capital and labour with a linear homogenous production function of the same form. This leads to the following resource constraints:

$$a_{1t} + a_{2t} = Y_{1t} = z_{1t}f(k_{1t}, n_{1t}) \qquad \text{(for country 1)}$$
$$b_{1t} + b_{2t} = Y_{2t} = z_{2t}f(k_{2t}, n_{2t}) \qquad \text{(for country 2)}$$

The quantity of Y_{1t} denotes output in country 1. Country 1 keeps a_{1t} units for domestic use and exports the rest a_{2t}. It then imports b_{1t} units from country 2 and combines them to make $G(a_1, b_1)$ units of country 1 specific good. Consumption, investment and government purchases are composite of foreign and domestic goods i.e.,

$$C_{1t} + I_{1t} + G_{1t} = G(a_{1t}, b_{1t})$$
$$C_{2t} + I_{2t} + G_{2t} = G(a_{2t}, b_{2t})$$

where $G(a,b) = [\omega a^{1-\alpha} + (1-\omega)b^{1-\alpha}]^{\frac{1}{1-\alpha}}$

Note that G is the Armington aggregator function (used to account for the finite elasticity of substitution between domestic and foreign goods) and ω_i are the weights associated with the shares of foreign and domestic goods. With $\omega = 1$ consumer's are indifferent between home and imported goods.

1. Derive an expression for trade balance?

2. Derive an expression for the real exchange rate defined as the relative price of imports to exports?

Solution

1) In order to relate the expenditure component $(C_{1t} + I_{1t} + G_{1t})$ to national output, note that the Armington aggregator function expresses consumption as a function of a_{1t} and b_{1t}. Given that the aggregator function, G, is homogeneous of degree one, we have, in equilibrium (by exploring the Euler's theorem);

Theorem *If $Q = f(K,L)$ is a linearly homogeneous, then $K\frac{\partial Q}{\partial K} + L\frac{\partial Q}{\partial L} \equiv Q$. The Euler's theorem (when applied in the context of a production function) states that under condition of constant return to scale, if each input factor is paid the cost of its marginal product, the total product will be exhausted by the distributive shares for all the input factors.*

The CES utility function, like all linearly homogenous functions, display constant returns to scale and hence qualifies for the application of Euler's theorem.

$$\Rightarrow C_{1t} + I_{1t} + G_{1t} = G(a_{1t}, b_{1t})$$
$$\Rightarrow C_{1t} + I_{1t} + G_{1t} = MU_a a_{1t} + MU_b b_{1t}$$

(where MU denotes marginal utility)

$$C_{1t} + I_{1t} + G_{1t} = P_H a_{1t} + P_F b_{1t} \qquad 47.1$$

where P_H and P_F are prices of home and imported (foreign) goods respectively. Note that we have made use of the fact that in equilibrium marginal utilities would be equivalent to their respective price ratios.

Substituting the resource constraint (country 1) for a_{1t} in (47.1) yields:

$$\Rightarrow \; C_{1t} + I_{1t} + G_{1t} = P_H(Y_{1t} - a_{2t}) + P_F b_{1t}$$

Dividing throughout by P_H yields:

$$Y_{1t} = \left(\frac{(C_{1t} + I_{1t} + G_{1t})}{P_H} \right) + a_{2t} - \left(\frac{P_F}{P_H} \right) b_{1t}$$

This equation is nothing but the GNP identity: sum of domestic absorption and net exports.

The trade balance (nx_t) is given by:

$$nx_t = a_{2t} - \left(\frac{P_F}{P_H} \right) b_{1t}$$

2) In order to compute the real exchange rate, in equilibrium, terms of trade $\frac{P_F}{P_H}$ for country 1 can be computed from the marginal rate of substitution in the Armington aggregator function. The marginal rate of substitution i.e., the shape of the indifference curve is given by:

$$\Rightarrow G(a,b) = [\omega a^{1-\alpha} + (1-\omega) b^{1-\alpha}]^{\frac{1}{1-\alpha}}$$

$$\Rightarrow \frac{P_F}{P_H} = S = \frac{\partial G(a_{1t}, b_{1t})/\partial b_{1t}}{\partial G(a_{1t}, b_{1t})/\partial a_{1t}} \Rightarrow$$

$$\frac{\frac{1}{1-\alpha}[\omega a^{1-\alpha} + (1-\omega) b^{1-\alpha}]^{\frac{1}{1-\alpha}-1}(1-\alpha)(1-\omega) b^{-\alpha}}{\frac{1}{1-\alpha}[\omega a^{1-\alpha} + (1-\omega) b^{1-\alpha}]^{\frac{1}{1-\alpha}-1}(1-\alpha)\omega a^{-\alpha}}$$

$$\Rightarrow \frac{P_F}{P_H} = S = \left(\frac{1-\omega}{\omega} \right) \left(\frac{a}{b} \right)^{\alpha}$$

or

$$S = \left(\frac{1-\omega}{\omega} \right) (K)^{\alpha} \qquad \text{(where } K = \frac{a}{b}\text{)} \qquad 47.2$$

The elasticity of substitution σ is given by:

$$\sigma = \frac{\partial K}{\partial S} \times \frac{S}{K} = \frac{1}{\alpha(\frac{1-\omega}{\omega})(K)^{\alpha-1}} \times \frac{(\frac{1-\omega}{\omega})K}{K} \Rightarrow \sigma = \frac{1}{\alpha} \qquad 47.3$$

Substituting (47.3) in (47.2) yields expression for the real exchange rate:

$$\frac{P_F}{P_H} = \left[\left(\frac{1-\omega}{\omega} \right) \left(\frac{a_{1t}}{b_{1t}} \right)^{\frac{1}{\sigma}} \right]$$

To the extent that home and foreign goods are not perfect substitutes, σ will take some finite value. The lower the estimate of σ means less substitution between home and imported goods.