A DSGE model of banks and financial intermediation with default risk

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August 2011

Abstract

This paper takes the view that a major contributing factor to the financial crisis of 2008 was a failure to correctly assess and price the risk of default. In order to analyse default risk in the macroeconomy, a simple general equilibrium model with banks and financial intermediation is constructed in which default-risk can be priced. It is shown how the credit spread can be attributed largely to the risk of default and how excess loan creation may emerge due different attitudes to risk by borrowers and lenders. The model can also be used to analyse systemic risk due to macroeconomic shocks which may be reduced by holding collateral.

Keywords: Default, Financial crisis, Financial intermediation, Liquidity shortages, Risk.
JEL Classification: E44, E51, G12, G21, G33

1. Introduction

This paper takes the view that a major contributing factor to the financial crisis of 2008 was a failure to correctly assess and price the risk of default. As a result, mortgage loans were under-priced and the riskiness of the lending financial institutions was greatly under-estimated. We therefore develop a general equilibrium model of banks and financial intermediation in which default-risk can be priced. The emphasis on default risk rather than on the liquidity shortgages that arose as a result of the markets discovering that the risk of default was very much higher than previously thought distinguishes this paper from much of the post-crisis literature on monetary policy.

This paper is not, of course, the first since the financial crisis to emphasise the role of risk or even of default. In the next section we review previous responses to the financial crisis and highlight how this approach differs from this paper. In section 3 we derive a general equilibrium model with banks and government which is used to price default risk. An important feature of the model is the consequence of the non-bank public and the banks having different attitudes to risk. This implications of the model are discussed in section 4 together with the effects of alternative possible specifications of the model, including its usefulness in distinguishing between systemic and idiosyncratic risk. Some conclusions are drawn in section 5.

2.Previous responses to the financial crisis

Attempts to formulate models capable of analysing and explaining the financial crisis have taken a number of different approaches. A common approach is to focus on bank runs and to draw on the seminal paper of Diamond and Dybvig (1983) which develops a portfolio theory of banking. A conspicuous and dramatic sign of a banking crisis is a run on banks as depositors seek to withdraw their funds rather than risk losing them entirely if the bank collapses. As the consequence of a widespread withdrawal of deposits is to precipitate the bank's collapse, central bank support for the bank is often required. This may take the form of providing liquidity to the bank, or of guaranteeing deposits. Diamond and Dybvig's proposal to avoid this is through deposit insurance provided by government and funded through an optimal tax on all consumers that creates no distortions. Allen, Carletti and Gale (2008) show how open-market operations conducted by the central bank through the interbank market, and funded by a tax imposed by government, can remove the risk of bank runs and enable the economy to achieve its optimal solution. For further discussion, see Allen and Gale (2007) and Miller, Zhang and Li (2010). The assets in these models are risk free and so do not allow for the possibility of default. Curdia and Woodford (2010) allow for bad loans but these are generated exogenously and the risk of bad loans is not priced.

An alternative explanation of liquidity shortgages is the existence of credit constraints, see Kiyotaki and Moore (1997). This may be related to an older literature dealing with borrowing constraints. The problem of adverse selection implies that, faced with imperfect information about which borrowers are likely to default, even in equilibrium, lenders may not only ration credit, they may also charge them different loan rates. Charging different loan rates may itself affect the behavior of particular borrowers, and the riskiness of loans, as those who are willing to borrow at a high interest rate may, on average, be more willing to default and hence take the greatest risks. This gives rise to the problem of moral hazard. These issues, both of which are features of the financial crisis, were examined by Stiglitz and Weiss (1981) and later by Walsh (2003). The implication is that the loan rate affects the quality of the loan as well as the profits of the lender. The higher the loan rate, the riskier the loan. One way to reduce this riskiness is to monitor loans. However, this is costly and could reduce the lender's profits, see Williamson (1987) and Walsh (2003). In effect, in credit markets, borrowers control the information and may, therefore, be said to be the agents of lenders. A lack of information about the circumstances surrounding the borrower's decisions may consequently result in agency costs: adverse selection, moral hazard and monitoring costs. Bernanke and Gertler (1989) have analysed the role of agency costs in a simple overlapping generations neoclassical model. They found that the higher the net worth of the borrower, the lower the agency costs. As a result, the stronger balance sheets of banks that emerge in good times tend to stimulate investment.

Curdia and Woodford (2008, 2011) have addressed the issue of quantitative easing when monetary policy is constrained by a zero lower interest rate bound; see also the comment by Dellas (2011). Their model is an extension of the standard New Keynesian model in which a key feature is the assumption that there is a real resource cost for financial intermediaries of orginating and monitoring loans and this may be reduced by holding reserves. These costs cause a difference between borrowing and lending rates, i.e. a credit spread. Although the model provides an explanation for the credit spread, which is attributed to a real cost of creating loans and to bad loans, and it introduces default into the model, it makes default exogenous and perfectly predictable. A more realistic treatment of default is that it is random and unpredictable. The model proposed in this paper is a simplified version of that of Curdia and Woodford (2011) in which we abstract from certain features, including the New Keynesian aspects of price setting and the labor market.

Gertler, Kiyotaki and Queralto (2010) have developed a DSGE model of financial intermediation that focuses on liquidity risk and how perceptions of asset return risk, as well as government policy interventions, influence the degree of risk exposure that financial intermediaries choose. In particular, banks face credit risk arising from accepting firm equity in exchange for loans to firms. Crucially, their theory assumes that firms are able to transfer their risks to the banks as there is no default risk, and banks do not charge a risk premium on these loans despite the risks arising from fluctuations in the value of firm equity. Gertler and Kiyotaki (2010) employ a related model to show how disruptions to financial intermediation can induce a crisis that affects real activity. Financial frictions, the source of disruption, causes banks to divert the funds they obtained in the inter-bank market and results in constraints on bank balance sheets, and hence in the provision of credit. This limits expenditures on investment, and so affects real activity. The central bank can relieve these financial constraints by injecting liquidity into the banks, and by providing funds directly to the private sector. Although Gertler and Kiyotaki address problems that may arise in financial intermediation, they do not consider the important issue of default.

One measure of the severity of the financial crisis is the emergence of large spreads between overnight inter-bank lending rates and the London inter-bank offer rates (LIBOR). These spreads could be due either to liquidity shortgages - as stressed in many of the theories above - or to default risk. Support for the latter is provided by Taylor and Williams (2008) who find that the main factor explaining the rise in such spreads is increased counterparty risk as captured by credit default swaps.

3. The Model

Due to the complexities of the banking system with its many financial products and intermediaries, we propose only a simple stylized model based on an assessment of risk and return. It has three sectors: a combined household-firm sector (or non-bank private sector), a banking sector and a consolidated government-central bank. Households hold deposits to pay for consumption which is cash-in-advance. They can also hold government bonds on which they receive a risk-free rate of return, and they can borrow from banks at a rate of return that reflects the possibility that the household could default due to an exogenous income or wealth shock. Loans and bonds are one-period. Banks do not pay interest on deposits but they receive interest from loans to households. Banks hold reserves at the central bank and can borrow either from the non-bank sector or, if this is constrained by a liquidity shortage, from the central bank - in each case at the risk-free rate of return. The central bank can also hold government debt and receive the risk-free rate. The government issues bonds, lends to banks and holds bank reserves. It also consumes and levies lump-sum taxes. The model does not assume either asymmetric or, due to shocks, complete information.

3.1 Non-bank Private Sector

The non-bank private sector aims to maximize

$$\mathcal{U}_t = E_t \Sigma_{s=0}^{\infty} \beta^s U(c_{t+s}) \tag{1}$$

subject to their budget constraint

$$L_{t+1} - \Delta D_{t+1} + R_t^{\mathrm{f}} A_t + P_t y_t + \Pi_t = P_t(c_t + i_t) + P_t T_t + \Delta A_{t+1} + \phi_t(1 + R_t)L_t,$$

where L_t are one-period nominal bank loans outstanding at the start of period t which carry a nominal interest rate of R_t, D_t are nominal bank deposits which are required for cash-in-advance consumption purchases, implying that $D_t = P_t c_t$, $A_t = B_t^{\mathrm{b}} + B_t^{\mathrm{g}}$ is the sum of bank-issued bonds B_t^{b} and government nominal bonds B_t^{g}, both of which pay a risk-free nominal rate of return of R_t^{f}, y_t is output and income from production, c_t is consumption, i_t is investment, T_t is lump-sum taxes and P_t is the price level. In each period a random proportion $0 \leq \phi_t \leq 1$ of repayments on loans (principal and interest) is made, implying either partial default or a probability of default. Unexpected changes in ϕ_t are negatively correlated with shocks to income and the loan rate. Bank profits Π_t are exogenous to the non-bank private sector. The national income identity is

$$y_t = F(k_t) = c_t + i_t + g_t, \tag{2}$$

where k_t is the (physical) capital stock and g_t is real government expenditures. Capital accumulation is determined by

$$\Delta k_{t+1} = i_t - \delta k_t. \tag{3}$$

The non-bank private-sector's resource constraint can therefore be written as

$$\begin{aligned} L_{t+1} - P_{t+1}c_{t+1} + (1 + R_t^{\mathrm{f}})A_t + P_t F(k_t) \;&=\; P_t[k_{t+1} - (1-\delta)k_t] + P_t T_t \\ &\quad + A_{t+1} + \phi_t(1 + R_t)L_t. \end{aligned} \tag{4}$$

The Lagrangian is

$$\begin{aligned}\mathcal{L}_t \;=\; & E_t\{\Sigma_{s=0}^{\infty}\beta^s U(c_{t+s}) + \lambda_{t+s}[L_{t+s+1} - P_{t+s+1}c_{t+s+1} + (1+R^{\mathrm{f}}_{t+s})A_{t+s} \\ & + P_{t+s}F(k_{t+s}) - P_{t+s}(k_{t+s+1} - (1-\delta)k_{t+s}) - P_{t+s}T_{t+s} - A_{t+s+1} \\ & - \phi_{t+s}(1+R_{t+s})L_{t+s}]\}. \qquad (5)\end{aligned}$$

The first-order conditions with respect to consumption, capital, nominal loans and nominal financial assets, all for $s > 0$, are

$$\begin{aligned}\frac{\partial \mathcal{L}_t}{\partial c_{t+s}} &= E_t\{\beta^s U'(c_{t+s}) - \lambda_{t+s-1}P_{t+s}\} = 0 \\ \frac{\partial \mathcal{L}_t}{\partial k_{t+s}} &= E_t\{\lambda_{t+s}P_{t+s}[F'(k_{t+s}) + 1 - \delta] - \lambda_{t+s-1}P_{t+s-1}\} = 0 \\ \frac{\partial \mathcal{L}_t}{\partial A_{t+s}} &= E_t\{\lambda_{t+s}(1+R^{\mathrm{f}}_{t+s}) - \lambda_{t+s-1}\} = 0 \\ \frac{\partial \mathcal{L}_t}{\partial L_{t+s}} &= E_t\{\lambda_{t+s-1} - \lambda_{t+s}\phi_{t+s}(1+R_{t+s})\} = 0.\end{aligned}$$

Recalling that R^{f}_{t+1} is the risk-free rate and is known at time t, we obtain

$$\begin{aligned}\lambda_t &= \frac{\beta E_t U'(c_{t+1})}{E_t P_{t+1}} \\ \lambda_t &= E_t\{\lambda_{t+1}\frac{P_{t+1}}{P_t}[F'(k_{t+1}) + 1 - \delta]\} \\ \lambda_t &= (1+R^{\mathrm{f}}_{t+1})E_t\lambda_{t+1} \\ \lambda_t &= E_t[\lambda_{t+1}\phi_{t+1}(1+R_{t+1})].\end{aligned}$$

From the last two equations, the loan rate satisfies the no-arbitrage equation

$$\begin{aligned}E_tR_{t+1} - R^{\mathrm{f}}_{t+1} \;=\; & \frac{1}{E_t\phi_{t+1}}\{(1+R^{\mathrm{f}}_{t+1})(1 - E_t\phi_{t+1}) - Cov_t(\phi_{t+1}, R_{t+1}) \\ & - \frac{Cov_t[\lambda_{t+1}, \phi_{t+1}(1+R_{t+1})]}{E_t\lambda_{t+1}}\}, \qquad (6)\end{aligned}$$

where, from the first-order condition for consumption,

$$E_t\lambda_{t+1} = \frac{1}{E_t P_{t+2}}[\beta^2 E_t U'(c_{t+2}) - Cov_t(\lambda_{t+1}, P_{t+2})]. \qquad (7)$$

Equation (6) is the key equation for analyzing the consequences of taking account of the possibility of default on loans. It can be interpreted as determining the credit spread - the difference between the loan rate and the deposit rate - that the non-bank public is willing to pay. Each term on the right-hand side is positive. The first term implies that the lower is the proportion of the loan that is expected to be repaid (i.e. the greater the risk of default), the higher is the loan rate that is acceptable to the borrower. The second term is positive as a postive shock to the loan rate is negatively correlated with the proportion of the loan repaid. The last term is the usual component of the risk premium relating consumption to the cost of borrowing - in this case, the effective cost of borrowing after taking account of possible default. Taken together these three terms are, in effect, the risk premium on loans.

The higher is ϕ_{t+1}, the proportion of the loan repaid in period $t+1$, the smaller is the risk premium and, *cet. par.,* the less will be the demand for loans. An under-estimate of ϕ_{t+1}, arguably as happened in the financial crisis, would lead to loans being under-priced, and to an excess of loans. The second term implies that the risk premium is larger, the greater is the correlation between default and the loan rate. The last term represents the usual component of the risk premium associated with the correlation between consumption and the risky rate of return, but here adjusted for the possibility of default. If default is absent, and so $\phi_t = 1$, then

$$E_t R_{t+1} - R^{\mathrm{f}}_{t+1} = -\frac{Cov_t[\lambda_{t+1}, R_{t+1}]}{E_t\lambda_{t+1}}. \qquad (8)$$

Hence, only price risk remains.

The no-arbitrage equation for the required real return on capital, $r^k_{t+1} = F'(k_{t+1}) - \delta$, is

$$E_t(r^k_{t+1} - r_{t+1}) \simeq -\frac{Cov_t[\pi_{t+1}, r^k_{t+1}]}{1 + E_t\pi_{t+1}} - \frac{Cov_t\{\lambda_{t+1}, (1+\pi_{t+1})(1+r^k_{t+1})\}}{E_t\lambda_{t+1}} \quad (9)$$

where $r_{t+1} = R^{\mathrm{f}}_{t+1} - E_t\pi_{t+1}$ is the real rate of return and $\pi_{t+1} = \frac{P_{t+1}}{P_t} - 1$ is the rate of inflation. The right-hand side is the risk premium on the return to capital. This does not depend on default risk.

The Euler equation for consumption is

$$\frac{\beta E_t U'(c_{t+2}) E_t P_{t+1}}{E_t U'(c_{t+1}) E_t P_{t+2}}(1 + R^{\mathrm{f}}_{t+1}) = \frac{Cov_t(\lambda_{t+1}, P_{t+2})}{E_t\lambda_{t+1} E_t P_{t+2}}. \quad (10)$$

Due to the cash-in-advance constraint, consumption must be planned one-period ahead in this model. Equation (10) shows that the cash-in-adance constraint adds a risk term to the Euler equation that is associated with not knowing the future price level.

3.2 Banks

Bank profits are given by

$$\Pi_t = \phi_t(1 + R_t)L_t - L_{t+1} + B_{t+1} - (1 + R^{\mathrm{f}}_t)B_t + \Delta D_{t+1} - \Delta H_{t+1}, \quad (11)$$

where $B_t = B^{\mathrm{b}}_t + B^{\mathrm{bg}}_t$ is the sum of nominal bank borrowing from the non-bank private sector B^{b}_t, and net borrowing from the central bank at the risk-free rate B^{bg}_t. H_t are reserves held at the central bank at no interest. Thus banks make their profits solely from lending to the private non-bank sector. This can be leveraged by borrowing from the total non-bank sector. In practice banks may also borrow internationally. If UIP holds then the definition of B_t could be amended to include foreign borrowing without any other change. The lower rate on bank borrowing than on the loan rate, and the lending of deposits, are the sources of bank profits. The simultaneous lending and borrowing by the private non-bank sector from banks can be justified by assuming that the household-firm holds a diversified portfolio of risky and risk-free assets, or by noting that the principal borrowers will be firms who seek a real, even if risky, return on their physical capital. The balance sheet of banks, which combines money and credit policy, is

$$H_t + L_t = D_t + B_t, \tag{12}$$

where the total money supply is D_t, high-powered money is H_t, and $L_t - B_t$ is net credit extended by banks.

We assume that banks take deposits D_t and reserve requirements H_t as given, and choose L_{t+s} and B_{t+s} $(s > 0)$ to maximize

$$\mathcal{P}_t = E_t \Sigma_{s=0}^{\infty} (1 + i_{t,t+s})^{-s} V(\Pi_{t+s}), \tag{13}$$

where $V(\Pi_t)$ is the utility that banks derive from profits and $i_{t,t+s}$ is the forward rate at time t for R^{f}_{t+s}. Forward rates are known at time t and satisfy $i_{t,t+s} = E_t R^{\mathrm{f}}_{t+s}$ $(s > 0)$. Introducing a bank utility function for profits allows banks to have an appetite for risk. They may be risk-averse or risk-loving, and not simply risk-neutral when $V(\Pi_t) \propto \Pi_t$. The first-order conditions for nominal loans and nominal borrowing for $s > 0$ are

$$\begin{aligned} \frac{\partial \mathcal{P}_t}{\partial L_{t+s}} &= E_t[(1 + i_{t,t+s})^{-s} V'(\Pi_{t+s}) \phi_{t+s} (1 + R_{t+s}) - (1 + i_{t,t+s-1})^{-(s-1)} V'(\Pi_{t+s-1})] \\ &= 0 \qquad (14) \\ \frac{\partial \mathcal{P}_t}{\partial B_{t+s}} &= E_t[(1 + i_{t,t+s-1})^{-(s-1)} V'(\Pi_{t+s-1}) - (1 + i_{t,t+s})^{-s} (1 + R^{\mathrm{f}}_{t+s}) V'(\Pi_{t+s})]\} \\ &= 0 \qquad (15) \end{aligned}$$

For $s = 1$, equation (15) implies that

$$\begin{aligned} V'(\Pi_t) &= E_t[(1 + i_{t,t+1})^{-1} (1 + R^{\mathrm{f}}_{t+1}) V'(\Pi_{t+1})] \\ &= E_t V'(\Pi_{t+1}). \qquad (16) \end{aligned}$$

as $i_{t,t+1} = E_t R^{\mathrm{f}}_{t+1}$ and both are known at time t. The marginal value of profits is therefore a martingale, and is constant in steady-state; under risk neutrality $V'(\Pi_{t+s}) = 1$ for all s. Equation (14) implies that

$$E_t[V'(\Pi_{t+1})\phi_{t+1}(1 + R_{t+1})] = (1 + R^{\mathrm{f}}_{t+1})V'(\Pi_t),$$

implying that the required expected excess return on bank loans to the private non-bank sector is

$$\begin{aligned} E_t R_{t+1} - R^{\mathrm{f}}_{t+1} \;=\; & \frac{1}{E_t\phi_{t+1}}\{(1 + E_t R^{\mathrm{f}}_{t+1})(1 - E_t\phi_{t+1}) - Cov_t(\phi_{t+1}, R_{t+1}) \\ & - \frac{Cov_t[V'(\Pi_{t+1}), \phi_{t+1}(1 + R_{t+1})]}{V'(\Pi_t)}\}. \end{aligned} \tag{17}$$

If there is no default risk then

$$E_t R_{t+1} - R^{\mathrm{f}}_{t+1} = -\frac{Cov_t[V'(\Pi_{t+1}), R_{t+1}]}{V'(\Pi_t)}.$$

If banks are risk-neutral then the last term in equation (17) is zero, and if, in addition, there is no default risk, the risk premium on loans is zero.

Comparing equations (6) and (17), they differ only in the last component of the risk premium. Loan market equilibrium requires that the two expressions are the same when

$$\frac{Cov_t[\lambda_{t+1}, \phi_{t+1}(1 + R_{t+1})]}{E_t\lambda_{t+1}} = \frac{Cov_t[V'(\Pi_{t+1}), \phi_{t+1}(1 + R_{t+1})]}{V'(\Pi_t)},$$

which holds if

$$\lambda_{t+1} + \varepsilon^{\lambda}_{t+1} = V'(\Pi_{t+1}) + \varepsilon^{\Pi}_{t+1}, \tag{18}$$

where $\varepsilon^{\lambda}_{t+1}$ and ε^{Π}_{t+1} are serially independent random variables having zero conditional means and a zero conditional covariance with $\phi_{t+1}(1+R_{t+1})$. This implies that $E_t\lambda_{t+1} = E_t V'(\Pi_{t+1})$, and hence that $\lambda_t/(1 + i_{t,t+1}) = V'(\Pi_t)$. The household's expected marginal utility from an additional unit of bank profit then equals its expected marginal utility for banks. This is a necessary condition for complete markets and a consequence of introducing a utility function for banks.

If instead banks and the non-bank private sector differ in their attitudes to risk then equations (6) and (17) would also differ. If banks were risk-neutral, and simply maximized profits, then the last term of equation (17) is zero. As the remaining terms in equations (6) and (17) are identical, this implies that the loan rate charged by banks, and obtained from equation (17), would be less than that which the non-bank public would be willing to pay. There would, therefore, be an excess demand for loans and credit rationing, but not necessarily adverse selection or moral hazard as both the lender and the borrower are aware of the risk of default. In contrast, if banks were more risk-averse than the non-bank public, then there would be an excess supply of loans.

3.3 Government

The model is completed with the consolidated government-central bank budget constraint which is

$$P_t g_t + (1 + R_t^{\mathrm{f}})(B_t^{\mathrm{g}} + B_t^{\mathrm{bg}}) = P_t T_t + B_{t+1}^{\mathrm{g}} + B_{t+1}^{\mathrm{bg}} + \Delta H_{t+1}. \qquad (19)$$

The government finances any deficit by borrowing B_t^{g} from the non-bank private sector at the official interest rate R_t^{f}. It may also sell debt to the banks or lend to them the net amount B_t^{bg} at this rate.

4. Discussion of the model

General equilibrium in this model entails the loan rate set by banks - equation (17) - is the same as that faced by the non-bank private sector - equation (6). This requires that the non-bank private sector and the banks have the same attitude to risk. The demand for loans by the non-bank private sector then equals the supply of loans by banks. Differences in their appetites for risk could lead to credit rationing if banks have a lower appetite, or to an excess supply supply of loans if banks are more risk averse. Changes in the underlying risk perceptions by banks may alter the loan rate and hence the volume of loans. For example, a reduction in the expected proportion of loans that are repaid in full would cause an increase in the loan rate, a reduction in the volume of loans next period and of bank profits in the current period.

The model assumes that default risk is correctly priced. If banks underestimated the risk of default then this would cause them to set too low a loan rate and, if the non-bank private sector correctly assessed their probability of default, the value of loans would be greater than it would be if the banks correctly estimated the default rate. It would also encourage banks to have an excess level of leverage of these loans; banks borrow at a low rate in order to lend at a higher rate, but they may over-borrow.

The model has a number of assumptions that have been made for their convenience. Because banks tend to borrow short and lend long, their profits are the result of being able to borrow at lower rates than the non-bank private sector. The model has assumed that all loans are for one period. It would be straightforward to assume instead that loans are long term. The term structure of interest rates could then be used to link the loan rate facing the non-bank private sector to the one-period loans assumed. This would introduce another source of risk, the term premium.

The model implies that the loan rate will exceed the deposit rate if there is a possibility of default - even partial default. This credit spread is likely to be greater, the less competitive is the banking sector. Given the concentration of banking in many countries, it seems more likely that banks also earn monopoly profits on credit spreads.

We have not taken into account the existence of the interbank market which played an important role in the financial crisis. The main function of the interbank market is to provide liquidity to the banking system by making over-night loans. This assists in smoothing the effects of lumpy transactions. These transactions involve short-term mismatches of flows into and out of banks, such as deposit flows. Banks can also borrow short-term from the interbank market to cover loan mismatches. As banks borrow short and lend long, they need to refinance large sums on a frequent basis. In the financial crisis, the interbank market was unwilling to fulfil this role due to extreme uncertainty about the risk of default of the banks themselves which created the liquidity crisis. Instead of taking account of the interbank market, we have consolidated the whole banking system. As a result, we are unable to capture this important feature of the financial crisis. It would, however, be straightforward to disaggregate the banking system and introduce interbank loans together with the risk of default on these loans; see, for example, Goodhart, Osorio and Tsomocos (2009). The greater the risk of default by a bank, the higher the cost of borrowing on the inter-bank market.

We have assumed that the default rate depends on macroeconomic shocks, for example, income shocks or shocks to the loan rate. This makes the default rate endogenous to the model. Default is most likely to occur when the net present value of an investment project is negative, when it is cheaper to default than to repay the loan principal and the interest. Thus default is likely to depend on factors like the sensitivity of the net wealth of the non-bank sector to shocks such as the ratio of loans to assets - a stock effect - and of the ratio of interest payments to income - a flow effect. Default may or may not result in bankruptcy due to negative net wealth. Partial default and a restructuring of the terms of the loan is often a better alternative for both parties than complete default.

When originating loans, the banks appear to have assumed that default was idiosyncratic, and so by holding a diversified portfolio it was possible to reduce risk. In the event, default proved to be much more systemic, implying that portfolio diversification would be far less effective in reducing risk. This distinction could be represented in the model by defining $\phi_t = \phi_t^I + \phi_t^S$, where ϕ_t^I is idiosyncratic risk and ϕ_t^S is systemic risk. In this way the default risk premium may be decomposed into two terms, one involving idiosyncratic risk and the other, much larger, representing systemic risk. ϕ_t^S will be affected by system-wide macroeconomic shocks. These will tend to strengthen the negative conditional covariance of ϕ_t^S with the loan rate R_t and hence raise the risk premium. In contrast, ϕ_t^I will not be much affected by macroeconomic shocks.

We have also omitted the crucial role of collateral: capital used to guarantee some repayment of a loan when defaulting. Collateral can be 100 per cent of the loan or less. The model can easily take account of collateral by amending the definition of ϕ_t, the proportion of the loan and interest repaid. The higher the rate of collateral, the higher the value of ϕ_t. In particular, collateral will reduce idiosyncratic risk, i.e. raise ϕ_t^I. In the absence of collateral, and faced with a low expected value of ϕ_t, credit will cost more and may even be refused entirely. A household or firm is then credit constrained. The reasons that credit-card debt costs so much more than other forms of borrowing are that it is not collateralized and that banks, through their own choice, have little knowledge of the financial circumstances of the card-holder. Not surprisingly, banks assume a high rate of default for credit-card debt.

In practice, the bulk of household loans are mortgages for house purchase. Houses are a durable and not a non-durable good, whereas the model assumes that consumer expenditures are solely for non-durables. Mortgages are taken out for many periods (several years) rather than for one period as in the model. Further, mortgage default and methods of mortgage finance by banks created the problems that led to the financial crisis. It is straightforward to extend the model to incorporate housing and mortgage finance. This would entail including durables in the household sector, and including longer maturity debt both in the non-bank public's and the banks' budget constraints. The cost of this debt can then be related to one-period debt via the term structure. The risk premium on mortgage debt will reflect the risk of default as in the model. Failure to price mortgage risk correctly was a key factor in bringing about the financial crisis.

The model describes the relations required for the economy to be in equilibrium when there is a possibility of default. Given the assumptions, a financial crisis is not predicted if the probability of default is evaluated correctly. In particular, the solution we have described assumes that the non-bank private sector and the banks agree on their assessment of the default rate. If they differ in this assessment then this would affect the solution. A crucial feature of the financial crisis is that the banks under-estimated the risk of default. In terms of the model this could happen if the wrong information were used to form expectations. In particular, the wrong distribution of shocks may have been used. A frequent comment made after the crisis was that the sequence of events that caused the crisis were so improbable that ignoring them was justified. The implication is that risks were correctly priced but, by chance, an extreme event occurred. An alternative interpretation is that, if the risk models being used predicted these events to be highly unlikely, then it is more probable that the model was wrong. Another possibility is that these were what is sometimes called Black Swan events where it is not the known unknowns that are a problem (i.e. extreme outliers whose possibility is known about), but the unknown unknowns (i.e. unimaginable events). Even if default risk is taken into account, there is therefore no guarantee that this would eliminate future crises.

5. Conclusions

The evidence, while not plentiful, indicates that default risk is the main contributory factor in explaining the financial crisis as measured by spreads between inter-bank lending and LIBOR. Much of the theory emphasises instead the role of liquidity. In this paper we take the view that liquidity shortages were a consequence of the inability of lenders to correctly assess and price the risk of default. The main contribution of this paper is the construction of a simple general equilibrium model of banks and financial intermediation in which default-risk can be priced.

We show that a credit spread - the difference between the loan rate and deposit rate (or the risk-free rate) - largely reflects the risk of default. The spread may also be affected if the non-bank private sector, the principle borrower, has a different attitude to risk from the banks, the main provider of loans. This may also result in excess loan creation. We show that the model can easily be adapted to analyse the systemic risk as well as idiosyncratic risk. We argue that systemic default risk is largely due to macroeconomic shocks. The model can also be re-interpreted to show why higher collateral is likely to reduce the risk of the idiosyncratic risk of default.

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