

# The Optimal Interest Rate on Reserves<sup>\*</sup>

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*Preliminary and Incomplete*

## Abstract

TBC.

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# 1 Introduction

Should the interest rate on reserves held at the Federal Reserve be set equal to the fed funds rate, eliminating the tax on the transactions balances that banks need to manage the liquidity of their deposits? The celebrated Friedman Rule says that there should be satiation in transactions balances since these balances are costless to produce. Operationally, the Rule says that the interest rate spread – or spreads – representing the opportunity cost of holding transactions balances should be set equal to zero.

Traditional discussions of the Friedman Rule (including Friedman (1959)) abstracted from the banking sector, and the opportunity cost of holding the transactions balances used by households and firms was typically taken to be the T-bill rate minus the rate of return on money (which is of course zero). A positive bond rate taxes money balances, and was usually referred to as *the* seigniorage tax, once again abstracting from the seigniorage tax on bank reserves. Canzoneri et al. (2010) surveyed a large literature that came to the following conclusion: the Friedman Rule is optimal if wages and prices are flexible, but it is not optimal in the presence of nominal price rigidities. The problem with the Friedman Rule is that it calls for a deflation (when real interest rates are positive), while the inefficiency created by price rigidities can only be eliminated by holding the aggregate price level constant. The Ramsey Planner had to tradeoff Friedman’s original monetary distortion and the distortion created by price rigidity. As documented in Canzoneri, Cumby and Diba’s survey, the tradeoff in most calibrated models was resolved decidedly in favor of near zero inflation, in violation of the original Friedman Rule.

But this well established literature does not necessarily imply that the Ramsey Planner will also want to tax bank reserves. In this paper, we ask whether it might still be optimal to eliminate the second seigniorage tax, the one on reserves, even if price rigidities call for taxation of money balances? We call this the Partial Friedman Rule: a positive seigniorage

tax on cash balances, but no seigniorage tax on reserves.

As far as we are aware, there has not been much discussion of the merits of the Partial Friedman Rule in the theoretical public finance literature. Curdia and Woodford (2011) argue that there should be no tax on reserves in a sticky price setting. But bank reserves are the only transactions balances in their model; they abstract from money balances that pay no interest. So, their result does not speak directly to the issue. In the applied policy literature, Goodfriend (2002) and others argue for satiation in bank reserves, but they had other motivations in mind. We will touch on their view briefly, but it does not speak directly to the issue either. Benigno and Nistico (2013) ... (TBC). Kashyap and Stein (2012) set out a three period model with an overlending externality; then, they argue that the fed funds rate might be focused on macroeconomic stability while the spread between the fed funds rate and the interest on reserves is used as a time varying Pigovian tax to offset the financial distortion. This paper may be the closest to ours, but it does not fill out the role played by the fed funds rate and the original seigniorage tax.

Here, we argue that the Full Friedman Rule (setting both seigniorage taxes to zero) is optimal if prices are flexible, but that neither it nor the Partial Friedman Rule is optimal in the presence price rigidities. In our simple model, banks need reserves to manage the liquidity of their deposits, and households need bank deposits, in addition to cash, to buy goods. So, a tax on reserves becomes a tax on deposits that distorts household consumption decisions.<sup>1</sup> When prices are flexible, the Full Friedman Rule eliminates all of the monetary distortions. But when we add price rigidities, the Ramsey planner can not manipulate the fed funds rate and the interest rate on reserves to eliminate all of the distortions. The optimal tax package includes a tax on deposits, which is obtained by a tax on reserves. In a parameterized version of the model, we also show how the Ramsey planner responds to

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<sup>1</sup>For simplicity, we abstract from bank deposits held by firms and other financial institution. As we shall see, the household consumption distortion provides a very easy way to illustrate the tradeoffs that the Ramsey Planner faces. However, this simplification minimizes the importance of the second seigniorage tax plays in the actual economy.

real shocks and to financial market shocks, and we compare the Ramsey solution to one that would prevail under a simple Taylor Rule and no interest on reserves).

In practice, central banks do not set their policy rates equal to zero during normal times, but many central banks do pay interest on reserves. At one extreme, the Reserve Bank of New Zealand has essentially eliminated the second seigniorage tax and driven reserves to satiation. At the other extreme, the Federal Reserve did not even have the authority to pay interest on reserves before 2008. The Fed had argued that it should be allowed to pay interest on reserves, but Congress did not want to take the budgetary hit. The Fed now pays interest on reserves, but it has kept the federal funds rate near zero since 2009, so we don't really know how the Fed will use the interest rate on reserves in normal times.<sup>2</sup> Many central banks have instituted a "corridor" system, one component of which is a small spread between the policy rate and the rate on reserves.<sup>3</sup> Our results provide a rationale for their doing so, but they may have other reasons in mind; see for example the discussions in Berentsen and Monnet (2008), Ennis and Keister (2008) and Bech and Klee (2011).

## 2 The Model

The basic economic environment is fairly simple to describe. There are three consumption goods: a cash good, a deposit good, and a credit good. Households face a cash in advance constraint to purchase the cash good, and a bank deposit in advance constraint to purchase the deposit good; households may pay for the credit good at the beginning of the next period. Firms face a wages in advance constraint before production begins; they can only satisfy this constraint by borrowing from commercial banks. Banks issue deposits and equity to fund their loans to firms, and they face a known default rate when making these loans; and banks

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<sup>2</sup>In the near term, the Fed has suggested that increasing the interest on reserves may be part of its exit strategy from the unconventional policy it has pursued during the Great Recession. (Need a Bernanke cite.) But, we are unaware of any statements about what will happen in the future.

<sup>3</sup>Examples include the ECB, the Bank of England and the Bank of Canada. Cite?

use central bank reserves or federal funds to manage the liquidity of their deposits. The central bank has two instruments of monetary policy: the federal funds rate and interest rate it pays on commercial bank reserves.

Each period is divided into two subperiods: a financial exchange followed by a goods exchange. In the financial exchange, after the realization of current shocks (including the loan default rate), firms borrow from banks to pay their wage bills. Households pay their taxes, they pay for the credit goods they purchased in the previous period, and they choose their asset portfolios, acquiring the money and deposits that they will need in the goods exchange. In the goods exchange, households use money and deposits to buy the cash and deposit goods.

## 2.1 Households

The representative household gets utility from consumption and disutility from work.

$$U_t = E_t \sum_{j=0}^{+\infty} \beta^j u(c_{t+j}^m, c_{t+j}^d, c_{t+j}^c, h_{t+j}) \quad (1)$$

where  $h_t$  is hours worked, and  $c_t^m$ ,  $c_t^d$ , and  $c_t^c$  are respectively the cash, deposit and credit goods. The household's budget constraint for the financial exchange, in real terms, is

$$\begin{aligned} \left(\frac{I_{t-1}^b}{\Pi_t}\right) b_{t-1}^H + \left[\left(\frac{I_{t-1}^d}{\Pi_t}\right) d_{t-1}^H - \frac{c_{t-1}^d}{\Pi_t}\right] + \left[\left(\frac{1}{\Pi_t}\right) m_{t-1}^H - \frac{c_{t-1}^m}{\Pi_t}\right] + w_t h_t + \frac{\Omega_{t-1}^F}{\Pi_t} + \frac{\Omega_{t-1}^B}{\Pi_t} \\ \geq \frac{c_{t-1}^c}{\Pi_t} + T_t + b_t^H + m_t^H + d_t^H, \end{aligned} \quad (2)$$

where  $\Pi_t \equiv \frac{P_t}{P_{t-1}}$  is the gross rate of inflation. The household's assets include CCAPM bonds ( $b_t^H$ ) with a gross nominal interest rate  $I_t^b$ , deposits ( $d_t^H$ ) with a gross nominal interest rate  $I_t^d$ , and money balances ( $m_t^H$ ) with no interest income. In the budget constraint,  $w_t$  is the real wage,  $T_t$  is a lump-sum tax,  $\Omega_t^F$  is the profits of firms, and  $\Omega_t^B$  is the dividends paid by

banks.

The household maximizes utility subject to its budget constraint and the cash and deposit constraints

$$m_t^H \geq c_t^m \quad (3)$$

$$d_t^H \geq c_t^d \quad (4)$$

Letting  $\lambda_t$  represent the Lagrange multiplier on the period  $t$  budget constraint, the household's first order conditions include

$$\frac{\partial u}{\partial c_t^m} = \lambda_t \quad (5)$$

$$1/I_t^b = \beta E_t \left\{ \frac{\lambda_{t+1}/\lambda_t}{\Pi_{t+1}} \right\} \quad (6)$$

$$\frac{\partial u}{\partial c_t^m} = I_t^b \frac{\partial u}{\partial c_t^c} \quad (7)$$

$$\frac{\partial u}{\partial c_t^d} = [1 + (I_t^b - I_t^d)] \frac{\partial u}{\partial c_t^c} \quad (8)$$

$$\frac{\partial u}{\partial h_t} = w_t \frac{\partial u}{\partial c_t^m} \quad (9)$$

(5) says that the marginal utility of wealth,  $\lambda_t$ , is equal to the marginal utility of the cash good, and (6) is the standard Euler equation;  $I_t^b$  is the CCAPM rate at which households discount the future. Interpretation of next three conditions is straightforward once the timing of household payments is taken into account. (7) is the first order condition for the credit good; if the household gives up one dollar's worth of the cash good, it can spend  $I_t^b$  dollars on the credit good, because it can hold bonds instead of cash. Similarly, (8) is the first order condition for the deposit good; if the household gives up one dollar's worth of the deposit good, it can spend  $1 + (I_t^b - I_t^d)$  dollars on the credit good, because it can hold bonds instead of deposits. The final equation is the labor supply curve.

Letting  $\mu_t^m$  and  $\mu_t^d$  represent the multipliers on the cash and deposit in advance constraints, the complementary slackness conditions are

$$\begin{aligned}\mu_t^m (m_t^H - c_t^m) &= 0 \\ \mu_t^d (d_t^H - c_t^d) &= 0\end{aligned}\tag{10}$$

and the multipliers themselves are given by

$$\begin{aligned}\mu_t^m &= \left( \frac{I_t^b - 1}{I_t^b} \right) \lambda_t \\ \mu_t^d &= \left( \frac{I_t^b - I_t^d}{I_t^b} \right) \lambda_t\end{aligned}\tag{11}$$

(11) says that  $\mu_t^m$  is the marginal disutility of holding cash instead of bonds and  $\mu_t^d$  is the marginal disutility of holding deposits instead of bonds. If the interest rates on money, deposits and bonds were to equalize, the multipliers would fall to zero and the liquidity constraints would not be binding.

These equations illustrate the household distortions created by the two seigniorage taxes. The standard seigniorage tax  $- I_t^b - 1$  distorts the margin between cash goods and credit goods; when  $I_t^b = 1$ , the cash in advance is not binding. The second seigniorage tax  $- I_t^b - I_t^d$  distorts the margin between deposit goods and credit goods; when  $I_t^b = I_t^d$ , the deposit in advance constraint is not binding. We will see that the second seigniorage tax can come from a tax on reserves. These seigniorage taxes also distort the margins between work and

consumption; that is, the first order conditions imply

$$\begin{aligned}
\frac{\partial u}{\partial h_t} &= w_t \frac{\partial u}{\partial c_t^m} \\
&= w_t I_t^b \frac{\partial u}{\partial c_t^c} \\
&= w_t \left[ \frac{I_t^b}{1 + (I_t^b - I_t^d)} \right] \frac{\partial u}{\partial c_t^d}
\end{aligned} \tag{12}$$

Our assumption that workers get their wages during the period in which they work implies that the margin between work and cash good consumption is not taxed.

## 2.2 Banks, Money Markets and Financial Frictions

The banking sector is perfectly competitive. Banks choose their assets and liabilities to maximize the stock market value for their owners, the households. Banks make loans to producers of intermediate goods (as described below); these firms have to borrow from banks to finance their wage bills, and this is costly financial friction since banks incur costs in making loans. Banks themselves face no frictions in raising funds. So, the Modigliani-Miller Theorem (Modigliani and Miller (1958)) implies that debt and equity financing are equivalent; we will assume that banks use equity finance when their loans exceed deposits. The market value of banks is equal to the present value of their cash flows since we assume the timing of dividends is irrelevant; so, banks maximize the expected present value of their dividend stream, calculated using the household's stochastic discount factor. Finally, banks hold reserves to manage the liquidity of their deposits.

More specifically, the representative bank maximizes

$$E_t \left\{ \sum_{j=0}^{+\infty} \beta^j \left( \frac{\lambda_{t+j}}{\lambda_t} \right) \Omega_{t+j}^B \right\} \tag{13}$$



where cash flow,  $\Omega_t^B$ , is given by

$$\begin{aligned} \Omega_t^B = & \left[ d_t - \left( \frac{I_{t-1}^d}{\Pi_t} \right) d_{t-1} \right] + \left[ (1 - \delta_{t-1}) \left( \frac{I_{t-1}^l}{\Pi_t} \right) l_{t-1} - l_t \right] - \tau_t d_t \\ & + \left[ \frac{m_{t-1}^B}{\Pi_t} - m_t^B \right] + \frac{i_{t-1}^r}{\Pi_t} (m_{t-1}^B + f_{t-1}) - \left( \frac{i_{t-1}^f}{\Pi_t} \right) f_{t-1} \end{aligned} \quad (14)$$

The first two terms on the RHS of (14) arise from issuing and redeeming deposits and loans;  $I^l$  is the gross nominal interest rate on bank loans ( $l$ ). We assume that a fraction  $\delta_t$  of the borrowers experience a default shock and pay nothing back on their loans;  $\log(\delta_t)$  follows a stationary AR(1) process with mean  $\log(\bar{\delta})$ . The bank observes the default rate before it makes loans, but the bank doesn't know the identity of the borrowers who will ultimately default.

The bank also incurs transactions costs,  $\tau_t d_t$ , in managing the liquidity of its deposits; it bank can lower these costs by holding reserves. The bank's reserves at the central bank consist of already existing deposits ( $m^B$ ) and reserves it borrows on the interbank market ( $f$ ); the central bank sets the gross nominal interest rate  $I^r$  on reserves, and the market determines the gross return  $I^f$  on federal funds. The corresponding net interest rates are  $i^r$  and  $i^f$ . As shown in the last two terms on the LHS of (14), the bank receives  $i^r$  on all of its reserves at the central bank, but it has to pay  $i^f$  on the reserves it borrowed from other banks.<sup>4</sup>

So, what are the transactions costs,  $\tau_t d_t$ , that the bank faces in managing the liquidity of its deposits? In reality, banks need to hold reserves because of a potential mismatch in the maturity of its deposits and loans. In our simple model, there is no such mismatch. But to capture the demand for reserves, we assume the bank incurs transactions costs that are proportional to the amount of its deposits. We let the factor of proportionality,  $\tau_t$ , depend

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<sup>4</sup>In a symmetric equilibrium, there will be no net lending on the interbank market, but we want to determine the equilibrium federal funds rate.

on the velocity of deposits relative to reserves

$$\nu_t = \frac{d_t}{m_t^B + f_t} \quad (15)$$

and we adopt the functional form that Schmitt-Grohe and Uribe (2004) used in modeling transactions costs faced by households; letting  $\nu^*$  be the satiation level of velocity, we set

$$\tau_t = \begin{cases} (A/\nu_t)(\nu_t - \nu^*)^2 & \text{if } \nu_t > \nu^* \\ 0 & \text{otherwise} \end{cases} \quad (16)$$

where  $A$  is a positive cost parameter.<sup>5</sup> The bank can lower its transactions cost by holding more reserves; when velocity reaches its satiation point, the cost of managing deposits falls to zero.

The bank's optimality condition for own reserves  $m_t^B$  is

$$\frac{I_t^b - I_t^r}{I_t^b} = A [(\nu_t)^2 - (\nu^*)^2] \quad (17)$$

linking the opportunity cost of holding reserves (instead of issuing CCAPM bonds) to the marginal benefit in terms of reducing transactions costs.<sup>6</sup> If  $I_t^b > I_t^r$ , the bank will not accumulate enough own reserves to drive its transactions costs to zero. If the central bank sets the opportunity cost to zero – by letting the interest rate on reserves equal the bond rate – the bank will satiate its demand for own reserves. Similarly, the optimality condition for borrowed reserves  $f_t$  is

$$\frac{I_t^f - I_t^r}{I_t^b} = A [(\nu_t)^2 - (\nu^*)^2] \quad (18)$$

linking the marginal cost of holding borrowed reserves (instead of lending on the federal

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<sup>5</sup>See Schmitt-Grohe and Uribe's (Schmitt-Grohe and Uribe (2004) motivation for this particular functional form.

<sup>6</sup>The return,  $I_t^b - I_t^r$ , is not realized until next period, so it is discounted by  $I_t^b$ .

funds market) to the marginal benefit in terms of reducing transactions costs.

This last first order condition can be viewed as the total demand for reserves (or federal funds). Using the definition of velocity, the demand curve in period  $t$  can be written as

$$\frac{I^f - I^r}{I^b} = Ad^2 \left[ \left( \frac{1}{m + f} \right)^2 - \left( \frac{\nu^*}{d} \right)^2 \right]$$

for a given level of deposits,  $d$ . This demand curve is pictured in Figure 1. If the central bank sets the interest rate on reserves (possibly at zero), and open market operations make the total supply of reserves equal to  $RS_1$ , then the intersection of supply and demand determines the fed funds rate. If commercial bank deposits ( $d$ ) increase, the demand curve shifts to the right, and the fed funds rate will rise; the central bank can of course accommodate the increase in demand with an open market operation that increases supply, keeping the fed funds rate on target. This is the textbook view of the federal funds market. But as Goodfriend (2002) and others have noted, open market operations can be divorced from the fed funds target rate if the supply of reserves is pushed past the satiation point,  $d/\nu^*$ . At  $RS_2$ , the interest on reserves is equal to the fed funds rate, and open market operations can be used to pursue other goals (as long as they maintain satiation in reserves), such as providing extra liquidity for stressed financial institutions. We will not pursue this possibility in the present paper.

The bank's optimality condition for deposits is

$$I_t^d = [1 - 2A(\nu_t - \nu^*)] I_t^b \tag{19}$$

or alternatively

$$\frac{I_t^b - I_t^d}{I_t^b} = 2A \left( \frac{d_t}{RS_t} - \nu^* \right) \tag{20}$$

The bank can raise funds by issuing deposits or by issuing CCAPM bonds. But, if it issues

deposits, it has to hold reserves to manage their liquidity. If  $RS_t$  is too low for bank satiation in reserves, or equivalently, if the interest on reserves is less than the fed funds rate, then issuing deposits is costly, and the deposit rate will be lower than the bond rate.

Now, we can get a better understanding of the second seigniorage tax in our model. As explained above, the household has to needs deposits to purchase the deposit good, and this may distort its consumption decisions. More precisely, (8) shows that households will consume too little of the deposit good if  $I_t^b > I_t^d$ , or equivalently, if  $I_t^f > I_t^r$ . The latter spread represents a tax on reserves, and therefore an implicit tax on deposits and deposit good consumption. This is the second seigniorage tax will play an important role in the next section.

Finally, the bank's optimality condition for lending is

$$(1 - \delta_t) I_t^l = I_t^b \tag{21}$$

Banks can lend on the bond market, or they can lend to firms. When they lend to firms, they know that they face a default rate equal to  $\delta_t$ . This drives a wedge between the loan rate and the bond rate: performing loans must pay for the loans that fail. This represents a financial friction that cannot be eliminated by monetary policy. Firms have to borrow from banks, instead of issuing bonds, and this is an expensive way for firms to meet their wage bills.<sup>7</sup>

We have made two assumptions along the way that simplify our analysis considerably. First, the definition of velocity, (15), implies that own reserves ( $m^B$ ) and borrowed reserves ( $f$ ) are perfect substitutes in the management of bank deposits. And more importantly, the bank faces no frictions in funding itself; so, the bank is indifferent between borrowing on the interbank market or on the bond market. As a consequence, (18) and (17) imply that the

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<sup>7</sup>The only role of  $\delta_t$  in the present version of the paper is to add a financial market shock. In future versions, it may be the source of an externality that will lead to overlending.

fed funds rate and the bond rate must equalize; that is,  $I_t^f = I_t^b$  in equilibrium. If  $I_t^f$  were greater than  $I_t^b$ , then the bank would raise funds by issuing bonds and lend in the fed funds market; if  $I_t^b$  were greater than  $I_t^f$ , the bank would borrow on the federal funds market and lend in the bond market.

While these assumptions bring simplicity, they may also have eliminated an important component in the transmission mechanism for monetary policy. In particular, when (as we will assume) the central bank uses open market operations to set the fed funds rate, it is implicitly setting the rate of return on CCAPM bonds. There is no wedge between money market rates and the interest rate that goes into the Euler equation.<sup>8</sup>

## 2.3 Firms and the Demand for Loans

We will consider two cases: one with competitive firms and flexible prices, and another with monopolistic competition and price rigidities. In either case, the final output can be sold as a cash good, a deposit good or a credit good; thus, the marginal cost of producing the three consumption goods is exactly the same.

In our model with flexible prices, a unit mass of competitive firms produce the economy's final output. To generate a demand for bank loans, we assume that workers must be paid when the service is rendered, and that firms have to borrow from banks to pay their wage bills.<sup>9</sup> More precisely, firm  $i$  takes a bank loan  $l_t(i)$  in the financial exchange to pay its workers. After the financial exchange, a fraction  $\delta_t$  of these firms are hit by a default shock: they pay their workers in full, and the workers put in the required effort, but ultimately the firm does not manage to bring anything to market.

Firm  $i$  has a linear production technology

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<sup>8</sup>Elsewhere, we have shown that this may lead to some empirical difficulties; see Canzoneri et al. (2007).

<sup>9</sup>This is a standard way of modeling working capital. See for example Christiano et al. (2005).

$$y_t(i) = z_t h_t(i)$$

where  $\log(z_t)$  follows a stationary AR(1) process with zero mean. The firm sets its labor input to maximize profits

$$z_t h_t(i) - I_t^l w_t h_t(i)$$

taking the wage rate and the loan rate as given. The firm sets marginal cost equal to marginal product

$$I_t^l w_t = z_t \tag{22}$$

and earns zero profits in equilibrium.

In our model with price rigidities, we add monopolistic competition and price setting firms. We model firms in two layers. The first layer consists of the competitive firms described above: these firms take bank loans, and may experience a default shock. The new element is that the output of these competitive firms is now an intermediate good that is sold to a second layer of firms; the second level firms produce differentiated goods and set prices subject to a price adjustment cost.<sup>10</sup> The final output is the usual Dixit Stiglitz aggregate of these differentiated goods.

More precisely, a fraction  $\delta_t$  of the firms producing the intermediate good are hit by the default shock: they pay their workers but do not manage to produce anything. If firm  $i$  is in remaining fraction  $1 - \delta_t$  of intermediate good firms, it produces

$$x_t(i) = z_t h_t(i)$$

and sells the intermediate output at a price  $p_t^x$  to producers of differentiated goods. Firm  $i$

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<sup>10</sup>Similar two-layer setups are often used in models with financial frictions. In our model, this is helpful because we want to allow for a default shock on bank loans, but we don't want this shock to affect the number (unit mass) of varieties produced by the firms that have price rigidity.

maximizes

$$\left(\frac{p_t^x}{p_t}\right) z_t h_t(i) - I_t^l w_t h_t(i)$$

which implies

$$p_t^x = \frac{I_t^l p_t w_t}{z_t}$$

and zero profits.

To model the producers of differentiated goods, we use a standard framework with price adjustment costs. Firm  $f$  faces the demand function

$$y_t(f) = \left[\frac{p_t}{p_t(f)}\right]^\sigma y_t$$

and has the production technology

$$y_t(f) = x_t(f)$$

It converts one unit of the intermediate good into one unit of differentiated product,  $y_t(f)$ .

The firm takes the input price as given, and sets its own price to maximize

$$E_t \sum_{j=0}^{+\infty} \beta^j \left(\frac{\lambda_{t+j}}{\lambda_t}\right) \left\{ \frac{S p_{t+j-1}(f) y_{t+j-1}(f)}{p_{t+j}} - \frac{p_{t+j-1}^x y_{t+j-1}(i)}{p_{t+j}} \right. \\ \left. - \frac{\zeta}{2} \left[ \frac{p_{t+j}(i)}{p_{t+j-1}(i)} - 1 \right]^2 y_{t+j} \right\}$$

subject to its demand function.  $S (= \frac{\sigma}{\sigma-1})$  is a fiscal subsidy that eliminates the price markup

in the steady state.<sup>11</sup> The optimality condition for setting  $p_t(if)$  is

$$\beta E_t \left\{ \left( \frac{\lambda_{t+1}}{\lambda_t} \right) \left( \frac{S(1-\sigma) [p_t(if)]^{-\sigma} (p_t)^\sigma y_t}{p_{t+1}} + \frac{\sigma [p_t(f)]^{-\sigma-1} (p_t)^\sigma y_t I_t^l w_t}{z_t \Pi_{t+1}} \right) \right\} \\ - \zeta \left( \frac{p_t(f)}{p_{t-1}(f)} - 1 \right) \frac{y_t}{p_{t-1}(f)} + \zeta \beta E_t \left\{ \left( \frac{\lambda_{t+1}}{\lambda_t} \right) \left( \frac{p_{t+1}(if)}{p_t(if)} - 1 \right) \frac{p_{t+1}(f) y_{t+1}}{[p_t(f)]^2} \right\} = 0 \quad (23)$$

## 2.4 Macroeconomic Policy and Equilibrium

The central bank sets two seigniorage taxes; so, monetary policy clearly plays a role in government finance. However, we want to abstract from fiscal policy to the extent possible. To this end, we have set government purchases to zero, and we have assumed that the government can service its outstanding debt and pay for the subsidies with a lump sum tax (if the seigniorage taxes are not sufficient for these purposes). This simplification may however limit the importance of the seigniorage taxes and affect the quantitative results we derive in the next section.

In a symmetric equilibrium with sticky prices, (23) becomes a Phillips curve

$$\beta E_t \left( \frac{\lambda_{t+1}}{\lambda_t \Pi_{t+1}} \right) \left[ S(1-\sigma) + \frac{\sigma I_t^l w_t}{z_t} \right] = \zeta (\Pi_t - 1) \Pi_t \\ - \zeta \beta E_t \left\{ \left( \frac{\lambda_{t+1}}{\lambda_t} \right) (\Pi_{t+1} - 1) \Pi_{t+1} \left( \frac{y_{t+1}}{y_t} \right) \right\} \quad (24)$$

If prices are flexible, (22) replaces (24).

The central bank uses open market operations to set the fed funds rate, and it directly sets the interest rate on bank reserves,  $i_t^r$ . In equilibrium, we have

$$m_t^H + m_t^B = m_t$$

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<sup>11</sup>We don't want monetary policy to be able to exploit this markup.



But we don't need to keep track of monetary aggregates because monetary policy sets interest rates and accommodates money demand. Bank loans equal the wage bill in equilibrium

$$l_t = w_t h_t$$

Finally, the goods market clearing condition is

$$c_t^m + c_t^d + c_t^c + \varpi \tau_t d_t = \left[ 1 - \frac{\zeta}{2} (\Pi_t - 1)^2 \right] (1 - \delta_t) y_t \quad (25)$$

where  $\varpi$  is the fraction of transactions costs that are social costs, eating output that could otherwise be devoted to household consumption; the remainder of the transactions costs are transfers from the banks and their depositors (the households) in the bank's management of the liquidity of deposits.

## 2.5 Model parametrization

The parametrization is mostly standard, aside from the steady state value of the default parameter. We used .... The rest of parametrization is described in an appendix (which has yet to be written).

## 3 Optimal Monetary Policy

In this section, we characterize the Ramsey Planner's solution for two separate cases: in the first, firms are competitive and prices are flexible; in the second, firms that produce differentiated products face price adjustment costs. We will show that the Full Friedman Rule is optimal when prices are flexible. We are able to derive the Planner's solution analytically in this case. We will then show that the even the Partial Friedman Rule is not optimal when firms face price adjustment costs. We are not able to derive the Planner's solution

analytically in this case. We have to use a calibrated model to characterize the Planner's steady state solution, and Planner's response to real and financial shocks.

The Planner uses open market operations to set the fed funds rate, and it sets the interest on reserves directly. These are the instruments of monetary policy, but it will be recalled that the fed funds rate is equal to the bond rate in equilibrium in our model; that is,  $I_t^f = I_t^b$ . The Planner's goal is to maximize household utility subject to the optimality conditions of households (including the complementary slackness conditions), the optimality conditions of banks, the optimality conditions of firms, and the market clearing conditions. We can dispense with the government budget constraint, since the government has a lump sum tax available to it; we should note that this simplification may come at the expense of minimizing the importance of the seigniorage taxes in what follows.

First, we review the various costs and distortions in the model that the Ramsey Planner would like to eliminate:

$$\begin{aligned}
\frac{\partial u}{\partial c_t^m} &= I_t^b \frac{\partial u}{\partial c_t^c} \\
\frac{\partial u}{\partial c_t^d} &= [1 + (I_t^b - I_t^d)] \frac{\partial u}{\partial c_t^c} \\
\frac{\partial u}{\partial h_t} &= w_t \frac{\partial u}{\partial c_t^m} \\
&= w_t I_t^b \frac{\partial u}{\partial c_t^c} \\
&= w_t \left[ \frac{I_t^b}{1 + (I_t^b - I_t^d)} \right] \frac{\partial u}{\partial c_t^d} \\
c_t^m + c_t^d + c_t^c + \varpi \tau_t d_t &= \left[ 1 - \frac{\zeta}{2} (\Pi_t - 1)^2 \right] (1 - \delta_t) y_t
\end{aligned}$$

where

$$\frac{I_t^b - I_t^d}{I_t^b} = 2A \left[ \frac{d_t}{RS_t} - \nu^* \right] \quad (26)$$

$$\tau_t = (A/\nu_t) \left[ \frac{d_t}{RS_t} - \nu^* \right] \quad (27)$$

The first five equations show how the two seigniorate taxes distort household's work and consumption decisions. conditions for the credit good's margins with the cash good and the deposit good. If  $I_t^b > 1$ , then the standard seigniorage tax distorts the margin between cash goods and credit goods, and the margins between work and credit good consumption and deposit good consumption. The Planner controls this tax by setting the fed funds rate (since  $I_t^b = I_t^f$ ). If  $I_t^b > I_t^d$ , then the second seigniorage tax distorts the margin between deposit goods and credit goods, and also the margin between work and deposit good consumption. The Planner controls this tax by setting the supply of reserves,  $RS_t$  (or equivalently, the spread  $I_t^f - I_t^r$ ), as shown by (26). Next, the feasibility condition shows that the fraction  $\varpi$  of bank transactions costs eats up output that could otherwise be consumed by households; these costs are also due to the second seigniorage tax. The Planner controls these resource costs by setting the supply of reserves,  $RS_t$  (or equivalently, the spread  $I_t^f - I_t^r$ ), as shown by (27). Finally, if  $\zeta > 0$ , price adjustment costs also eat up resources. The Planner controls these costs by setting the fed funds rate to control inflation,  $\Pi_t$ .

We now provide an intuitive explanation of our basic results before proceeding to the formal derivations. When prices are flexible ( $\zeta = 0$ ), The Ramsey Planner has the tools to eliminate of the costs and distortions. The Planner does this by implementing the Full Friedman Rule  $-1 = I_t^r = I_t^f (= I_t^b)$  – which eliminates both of the seigniorage taxes. With flexible prices, this is the happy end of the story.

When price rigidities are present, the Planner faces some tradeoffs. The Full Friedman Rule implies negative rates of inflation (if real interest rates are positive), but the distortion

created by price adjustment costs can only be eliminated by setting inflation equal to zero. In the steady state, the real bond rate is pinned down by the household discount rate in the steady state, and the Planner can only raise steady state inflation by increasing the fed funds rate (and thus the bond rate). But, this distorts the margin for cash goods and credit goods, and also the margin between work and credit good consumption. If the Planner leaves the interest on reserves at the fed funds rate, then the deposit rate will equal the bond rate, and the margins for deposit goods will remain undistorted. But, this would put all of the onus on margins distorted by the standard seigniorage tax on bonds. Welfare losses can be lowered by spreading the distortions across all of the margins. Therefore, the Planner will want to use the second seigniorage tax, the tax on reserves, to distort the deposit good margins as well; to do this, the Planner will set the interest rate on reserves below the fed funds rate. Even the Partial Friedman Rule is not optimal.

In much of what follows, we will simplify notation by letting

$$I_t^f = I_t^b \equiv I_t$$

And we will use a separable utility function:  $u \equiv \phi_m \ln(c^m) + \phi_d \ln(c^d) + \phi_c \ln(c^c) - \frac{1}{1+\chi} h^{1+\chi}$ . In our numerical simulations, we will actually let  $\phi_m = \phi_d = \phi_c$ ; this symmetry will facilitate the illustration of our results.

### 3.1 With Flexible Prices, the Full Friedman Rule is Optimal

In this section, we show formally that the Full Friedman Rule is optimal in the perfectly competitive version of our model. We use (9), (22) and (21) to get

$$w_t = \frac{c_t^m (h_t)^\chi}{\phi_m} = \frac{(1 - \delta_t) z_t}{I_t} \tag{28}$$

We use the household's optimality conditions for cash and deposit goods to write

$$\frac{\phi_d}{c_t^d} = \frac{\phi_m}{c_t^m} \left[ 1 + \frac{1 - I_t^d}{I_t} \right]$$

The rest of our derivations will be more transparent if we use two alternative representations of the Ramsey problem for the case with  $\varpi > 0$ , and the one with  $\varpi = 0$ .

For  $\varpi > 0$ , we write

$$c_t^m = c_t^d \left( \frac{\phi_m}{\phi_d} \right) \left[ \frac{2A(v_t - \nu^*) I_t + 1}{I_t} \right]$$

using the bank's optimality condition for deposits. We will use this to eliminate the consumption of cash goods from the Ramsey problem. We can ignore the cash in advance constraint because our Ramsey policy sets interest rates and accommodates money demand. Similarly, we can eliminate the consumption of credit goods using

$$c_t^c = c_t^d \left( \frac{\phi_c}{\phi_d} \right) [2A(v_t - \nu^*) I_t + 1]$$

We use these (with  $\phi_m + \phi_d + \phi_c = 1$ ) to write the Ramsey objective function as

$$E_t \left\{ \sum_{j=0}^{+\infty} \beta^j \left[ \ln(c_{t+j}^d) + (\phi_m + \phi_c) \ln[2A(v_{t+j} - \nu^*) I_{t+j} + 1] - \phi_m \ln(I_{t+j}) - \frac{1}{1+\chi} (h_{t+j})^{1+\chi} \right] \right\}$$

But our Ramsey problem, under flexible prices, is actually a static optimization problem: given the optimal path of interest rates, the household's Euler equation is the only restriction on the path of inflation. So, we need not impose the Euler equation as a constraint, and we can write the objective function as

$$\ln(c_t^d) + (\phi_m + \phi_c) \ln \frac{2A(v_t - \nu^*) I_t + 1}{I_t} [2A(v_t - \nu^*) I_t + 1] - \phi_m \ln(I_t) - \frac{1}{1+\chi} (h_t)^{1+\chi}$$

We eliminate the cash good from (28) and write this constraint as

$$\frac{c_t^d}{\phi_d} [2A(v_t - \nu^*) I_t + 1] = (1 - \delta_t) z_t (h_t)^{-\chi} \quad (29)$$

We let  $\lambda_t^h$  denote the Lagrange multiplier on (29) in our Ramsey problem. The only other constraint that the household block of the model imposes on the Ramsey problem is

$$d_t \geq c_t^d \quad (30)$$

(we also need to keep in mind the associated complementary slackness condition). We let  $\lambda_t^d$  denote the Lagrange multiplier on the inequality constraint (30) in the Ramsey problem. The constraints implied by the optimality conditions of firms and banks are already incorporated in our derivations above. In particular, we can let the Ramsey planner directly set  $v_t$  and then use (17) to infer the optimal interest rate on reserves. The only remaining constraint is the equilibrium condition in the goods market; we eliminate the consumption of cash and credit goods and write this as

$$(1 - \delta_t) z_t h_t = c_t^d \left[ 1 + \left( \frac{\phi_m + \phi_c I_t}{\phi_d} \right) \left( \frac{2A(v_t - \nu^*) I_t + 1}{I_t} \right) \right] + \frac{\varpi A d_t}{v_t} (v_t - \nu^*)^2 \quad (31)$$

with Lagrange multiplier  $\lambda_t^y$ .

Since our Ramsey problem has no dynamics, we suppress the time subscripts in the optimality conditions for  $h$ ,  $c^d$ ,  $d$ ,  $v$ , and  $I$ , listed below in that order

$$(h)^x = \chi \lambda^h (1 - \delta) z (h)^{-\chi-1} + \lambda^y (1 - \delta) z \quad (32)$$

$$\frac{1}{c^d} + \left( \frac{2A(v - \nu^*) I + 1}{\phi_d} \right) \lambda^h = \lambda^d + \left[ 1 + \left( \frac{\phi_m + \phi_c I}{\phi_d} \right) \left( \frac{2A(v - \nu^*) I + 1}{I} \right) \right] \lambda^y \quad (33)$$

$$\lambda^d = \frac{\varpi A}{v} (v - \nu^*)^2 \lambda^y \quad (34)$$

$$\begin{aligned} \frac{(\phi_m + \phi_c) I}{2A(v - v^*) I + 1} + I \left( \frac{c^d}{\phi_d} \right) \lambda^h &= \left[ \frac{c^d (\phi_m + \phi_c I)}{\phi_d} \right] \varpi \lambda^y \\ &+ \frac{Ad}{(v)^2} [(v)^2 - (v^*)^2] \varpi \lambda^y \end{aligned} \quad (35)$$

$$\begin{aligned} \frac{2A(\phi_m + \phi_c)(v - v^*)}{2A(v - v^*) I + 1} - \frac{\phi_m}{I} + 2A(v - v^*) \left( \frac{c^d}{\phi_d} \right) \lambda^h \\ - \lambda^y c^d \left\{ \frac{\phi_c}{\phi_d} \left( \frac{2A(v - v^*) I + 1}{I} \right) - \frac{1}{(I)^2} \left( \frac{\phi_m + \phi_c I}{\phi_d} \right) \right\} \leq 0 \end{aligned} \quad (36)$$

and  $I \geq 1$  with complementary slackness.

A candidate solution in which the inequality constraint (30) is slack would imply  $\lambda^d = 0$ ,  $I^d = I$ , and  $v = v^*$ , by (34). Corresponding policies would set  $I^r = I$ , according to (17). We can check to see if such a policy in conjunction with the Friedman Rule ( $I = 1$ ) satisfies the optimality condition. By (29), we have

$$\frac{c^d}{\phi_d} = (1 - \delta) z (h)^{-x}$$

and plugging this in (31), we get  $h = 1$ . So,  $c^d = \phi_d (1 - \delta) z$ . Plugging these values into (32) and (33), we get  $\lambda^h = 0$  and  $\lambda^y = 1 / (1 - \delta) z$ . These values satisfy (35) and turn (36) into

$$- \left( 1 + \frac{1}{\phi_d} \right) (\phi_m + \phi_c) \leq 0$$

This confirms that our candidate solution is indeed the optimal policy as long as there is some social cost associated with managing the liquidity of deposits ; optimal policy satisfies the Friedman Rule and also satiates the demand for reserves.

For  $\varpi = 0$ , the liquidity management cost is a private cost to banks but has no resource cost associated with it. In this case, it is more convenient to formulate a version of the Ramsey problem that does not explicitly involve deposits and velocity– and show later

(after we have a candidate for optimal policy) that the equations involving those variables are satisfied. We use the household and firm optimality conditions to express the Ramsey problem as maximizing

$$-\chi \ln(h_t) + (\phi_m + \phi_c) \ln[1 + (I_t - I_t^d)] - \phi_m \ln(I_t) - \frac{1}{1 + \chi} (h_t)^{1 + \chi}$$

subject to

$$(h_t)^{1 + \chi} = \phi_d \left[ 1 + \left( \frac{\phi_m}{\phi_d} \right) \left( 1 + \frac{1 - I_t^d}{I_t} \right) + \left( \frac{\phi_c}{\phi_d} \right) (1 + I_t - I_t^d) \right]$$

with multiplier  $\eta_t$ . The optimality condition for  $I^d$  is

$$-\frac{\phi_m + \phi_c}{1 + I - I^d} + \eta \left[ \frac{\phi_m}{I} + \phi_c \right] \geq 0$$

and  $I - I^d \geq 0$  with complementary slackness. The optimality condition for  $h$  is

$$\frac{\chi}{h} + h^\chi = (1 + \chi) h^\chi \eta$$

The optimality condition for  $I$  is

$$\frac{\phi_m + \phi_c}{1 + I - I^d} - \frac{\phi_m}{I} + \eta \left[ \frac{\phi_m (1 - I^d)}{(I)^2} - \phi_c \right] \leq 0$$

and  $I - 1 \geq 0$  with complementary slackness. Evaluated at the Full Friedman Rule with  $I = I^d = 1$ , the constraint implies  $h = 1$ , and the optimality condition for hours gives  $\eta = 1$ . The other two conditions are satisfied as equalities, confirming that the Full Friedman Rule is optimal. The optimality conditions of the bank confirm that we have  $I^r = 1$  and  $v = v^*$ . The CIA and deposit constraints are slack, and we can pick the path of inflation to satisfy the household's Euler equation.



## 3.2 With Price Rigidities, even the Partial Friedman Rule is Not Optimal

The distortion created by price adjustment costs can only be eliminated by keeping prices constant; the monetary distortions can only be eliminated by imposing the Full Friedman Rule, and the negative inflation rate that it implies. This creates a welfare tradeoff, and the Ramsey Planner spreads the costs of these distortions optimally.

In what follows, we will compare the Ramsey solution with what would be obtained under a simple Taylor Rule:

$$i_t = \bar{i} + 1.5(\pi_t - \bar{\pi}) \quad (37)$$

where small letters represent net interest and inflation rates, and bars indicate steady state values. Here, the steady state solution is imposed by our parametrization; we have not chosen to be optimal in any sense other than that we have set  $\bar{\pi} = 0$ , eliminating the price adjustment costs in the steady state. We have set the interest rate on reserves equal to zero for this exercise.

With price rigidities, there is an additional state variable – inflation – which makes it difficult to calculate the optimal solution analytically. Here, we use numerical methods to calculate the optimal steady state and the optimal response to productivity shocks and default shocks.

### 3.2.1 The Ramsey Planner's Steady State

Table 2 reports our steady state results. The first column gives optimal steady state values when prices are flexible. The Planner implements the Full Friedman Rule –  $I^r = I^f = 1$  – in this case, and as explained above, this rule makes  $I^d = I^b = 1$ . The marginal utilities of consumption equalize across the three goods, and since we have set  $\phi_m = \phi_d = \phi_c$  in our log-linear utility function, consumption of the three goods in Table 2 is equal, and adds up

to supply (or employment, since  $z = 1$  in the steady state). And, of course, inflation is equal to minus the real rate of interest.

The second column of Table 2 reports the steady state results when price adjustment is costly. Here, the Ramsey Planner is pulled between deflating to eliminate the monetary distortions and holding the price level constant to eliminate the price adjustment costs. We see that in our parametrization the price adjustment costs are much more important than the monetary distortions. Steady state inflation is very close to zero; however, it is still negative, and output falls a little bit as a consequence.

The Planner has to raise the bond rate to bring inflation close to zero (since  $\frac{I^b}{\Pi} = \frac{1}{\beta}$  in the steady state), and the Planner does this by increasing the fed funds rate (which is equal to the bond rate in our model). But now, the intra temporal consumption smoothing comes unglued. If the Planner maintained satiation in reserves, then as explained above,  $I^r = I^f$  and  $I^d = I^b$ . The entire tax burden would fall the margin between cash goods and credit goods, both in consumption and in work effort.<sup>12</sup> To spread the distortions optimally, the Planner needs to tax the deposit good; the Planner does this by lowering the interest on reserves, which in turn lowers the deposit rate. In Table 2, we see that consumption of the deposit good is the same as it was in the case of flexible prices, while the consumption of credit goods is a little higher and the consumption of cash goods is a little lower.<sup>13</sup>

The third column of Table 2 reports the steady state results when price adjustment is costly, but policy is dictated by the Taylor Rule, and the interest rate on reserves is set equal to zero. Steady state inflation has also been set to zero; so, the price adjustment costs are eliminated. Steady state output is however slightly lower than with the Ramsey policy. This is because the bond rate is higher than the Ramsey bond rate, and this distorts the labor - leisure margins more than in the Ramsey solution. Furthermore, the interest rate on reserves has not risen to spread the monetary distortions to the deposit good. Consumption

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<sup>12</sup>In addition, (??) implies that all of the labor - leisure distortion would fall on the credit good.

<sup>13</sup>This also spreads the distortions in the labor - leisure margins.

of the three goods is consequently more dispersed than in the Ramsey solution.

### 3.2.2 The Ramsey Planner's Response to Real Shocks and Financial Market Shocks

As stated in Section 2, the logs of the productivity shock,  $z_t$ , and the default shock,  $\delta_t$ , follow stationary AR(1) processes; the processes are highly inertial, with auto-regressive coefficients of 0.9. Here, we study the Ramsey Planner's response to shocks, and compare them to the model's responses under the simple Taylor rule, (37).

#### Response to a Decrease in Productivity

Figure 2A illustrates the dynamic response to a persistent decrease in productivity. Starting with the response under a Taylor Rule (shown by the dashed lines), we see a pattern that is mostly familiar from the DSGE literature. The negative productivity shock raises marginal cost; so, inflation increases. Labor demand decreases, lowering the real wage (not shown) and employment. Bank loans decrease as the wage bill falls. The Taylor rule reacts to the rise in inflation by increasing the fed funds rate; this moderates the inflation somewhat, but the price adjustment costs remain. The bond rate is equal to the fed funds rate in our model; so, the bank loan rate rises (see (21)), making firms want to contract even further. Moreover, the spread between the fed funds rate and the rate on reserves increases; recall that the latter is fixed, so the spread increases by the full amount of the increase in the fed funds rate. This rise in the tax on reserves then increases the spread between the bond rate and the deposit rate (see equation (??)). These interest rate movements affect the two seigniorage taxes in the households' first order conditions for consumption. The tax on cash goods  $-I_t^b - 1$  rises the most, and consequently cash good consumption falls the most. The tax on deposit goods  $-I_t^b - I_t^d$  rises some, and deposit good consumption falls

less than cash good consumption, but more than credit good consumption.<sup>14</sup>

The Ramsey solution's response (shown in the solid lines) smooths these distortions in a number of ways. The fed funds rate does not rise nearly as much, and inflation hardly moves, virtually eliminating the price adjustment costs; employment and output fall less than with the Taylor Rule. Furthermore, all of the consumption margins are smoothed. The interest rate on reserves rises (not pictured). This together with the smaller increase in the fed fund rate lowers the tax on reserves, and the spread between the bond rate and the deposit rate rises much less. Thus, the seigniorage taxes on the cash good and the deposit good rise much less than with the Taylor Rule. Cash good consumption and credit good consumption converge on deposit good consumption.

### **Response to an Increase in the Default Rate**

Figure 2B illustrates the dynamic response to a sustained increase in the default rate. Once again, dashed lines give the response under the Taylor Rule, and solid lines give the response under the Ramsey Policy. Comparing Figures 2A and 2B, we see that the patterns are amazingly similar. There is a very simple reason for this similarity. The two shocks increase marginal cost in exactly the same way.

Why is this? For a given fed funds rate, and thus a given bond rate, the banks' first order condition for loans, (21), says that an increase in default rate will require a rise in the loan rate,  $I_t^l$ ; banks have to charge a higher rate on loans because they will have a higher proportion of non-performing loans. But then, the firm's marginal cost is just  $\frac{I_t^l w_t}{z_t}$ ; an increase in the loan rate is isomorphic to a decrease in productivity. From here on, the story is the same as for a negative productivity shock.

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<sup>14</sup>The solid and dashed lines for deposit good consumption virtually on top of one another.

## 4 Conclusion

We have shown that the Full Friedman Rule is optimal when firms are competitive and wages and prices are flexible. In this case, the interest rate on reserves should be raised to fed funds target. Demand for reserves is satiated, validating the practice of the Reserve bank of New Zealand. But when we add price rigidities, the Full Friedman Rule cannot eliminate all of the costs and distortions in the model, and the optimal tax package requires some use of both seigniorage taxes. In this case the interest rate on reserves is held below the fed funds rate, validating the practice's of central banks using the corridor system, such as the ECB, the Bank of England and the Bank of Canada.

In our numerical simulations, the differences between the Ramsey Planner's solution and the solution that would obtain under Taylor Rule (with no interest paid on reserves) are quantitatively small. However, this may be misleading because of a number of simplifications that we have made to ease the exposition of our basic results. In our model, only the households use bank deposits. This assumptions made the Planner's tradeoffs quite transparent, but it ignores the fact that firms and other financial institutions also use bank deposits and loans. Abstracting from these activities probably limits the damage done by the distortions associated with the two seigniorage taxes. Similarly, only firms borrow from banks in our model; a more extensive modeling of bank lending would probably increase the importance of the seigniorage taxes. Finally, we have assumed that the government can finance its operations with a lump sum tax. If we added government spending, and assumed that only distortionary taxes were available, then the two seigniorage taxes would play a role in balancing to government's budget. If the seigniorage taxes were needed for these purposes, then the Planner's tradeoffs would presumably be worse.

Future extensions may include:

- Lending externality where  $\delta_t$  depends on aggregate lending. Individual bank does not

see that its lending affects  $\delta_t$ .

- Gertler-Kareidi banks, to include bank funding frictions.
- Distortionary taxation, to increase the importance of the seigniorate taxes.

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Table 1: Parameters (TBC)



Table 2: Ramsey Steady State (for interest rates and inflation, percent per annum)

	Flexible Prices	Price Rigidities	Taylor Rule
fed funds rate (= bond rate)	0.0	3.8	4.0
interest on reserves	0.0	2.8	0.0
interest on deposits	0.0	2.6	1.4
inflation	-4.0	-0.0020	0.0
cash good consumption	0.3303	0.3282	0.3284
deposit good consumption	0.3303	0.3303	0.3295
credit good consumption	0.3303	0.3313	0.3317
employment	1.0000	0.9989	0.9986

Figure 1: Federal Funds Market

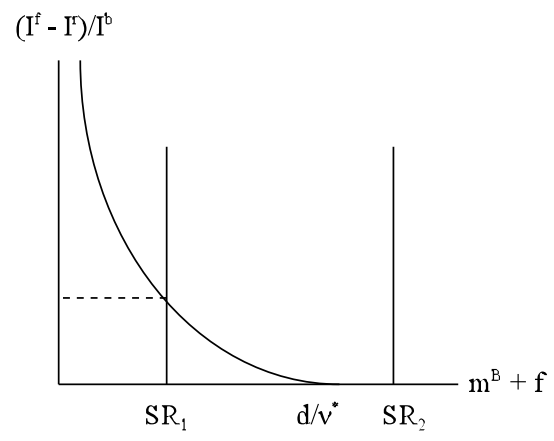
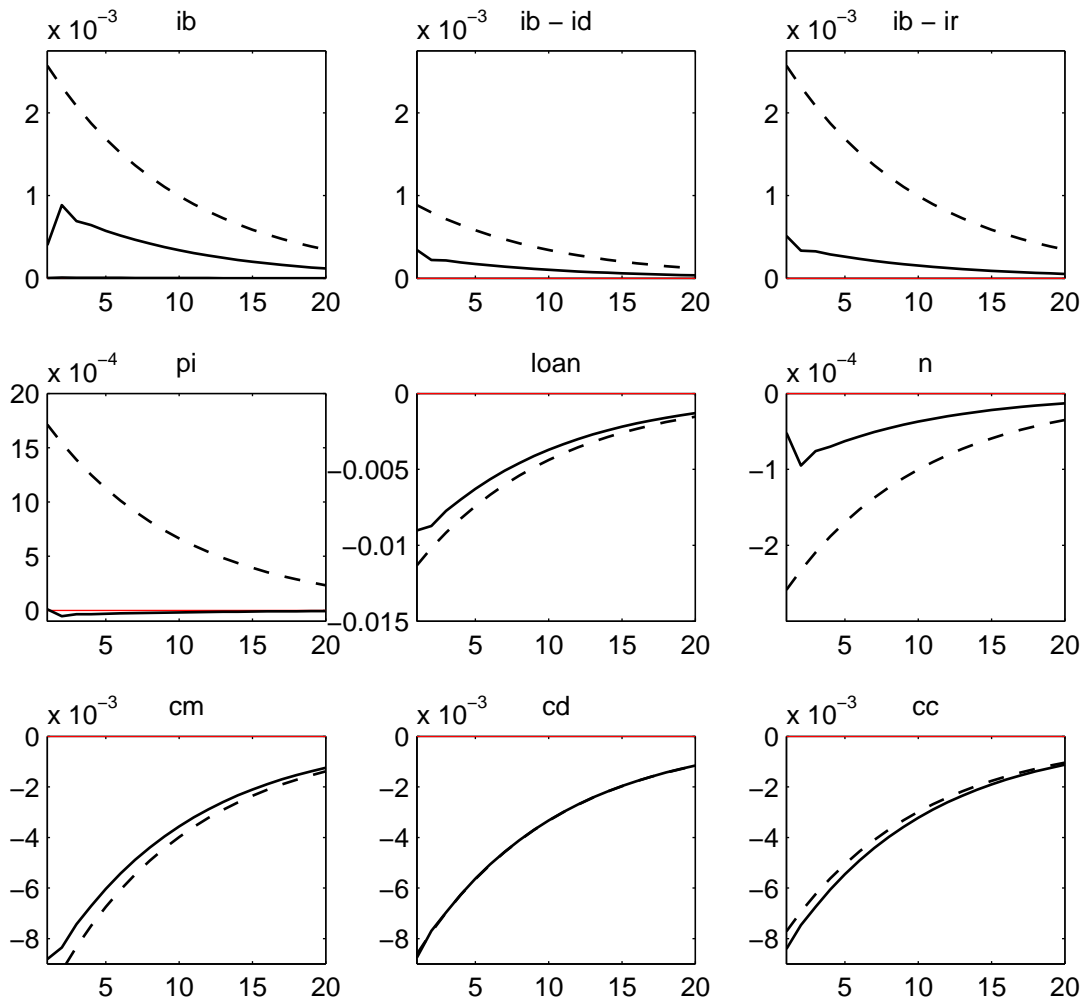
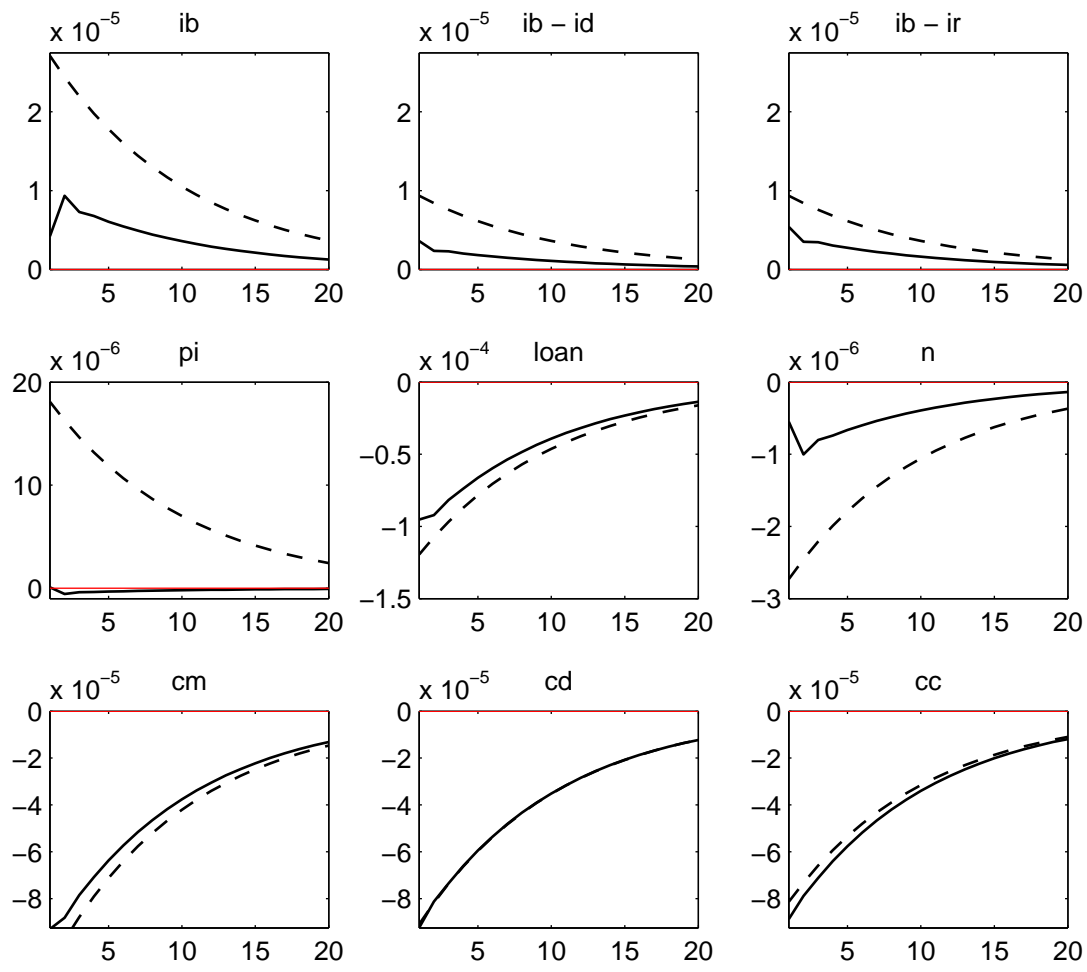


Figure 2A: Decrease in Productivity



Ramsey Policy = solid line; Taylor Rule = dashed line

Figure 2B: Increase in Default Rate



Ramsey Policy = solid line; Taylor Rule = dashed line