Inflation Inertia and Long-run Inflation Targeting in a Growing Economy: A Comparison of Calvo and Rotemberg

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Abstract

The observed 2% long run inflation target in most developed industrial nations is in variance with the zero or negative optimal inflation rates predicted by prominent monetary theories. Using a calibrated simple New-Keynesian model with endogenous growth and inflation inertia, we compare two price setting environments of Calvo (1983) and Rotemberg (1982). In our growth model, the steady state welfare maximizing inflation takes into account the growth effect as well as the price distortionary effects of inflation. The long-run welfare maximizing trend inflation could be positive in economies with inflation inertia. A higher degree of inflation indexation (lower inflation inertia) lowers the steady state price distortion in the Calvo model and steady state price adjustment cost in Rotemberg model and raises the long run optimal inflation. Since the productive inefficiency caused by inflation inertia is higher in Calvo economy compared to Rotemberg, the long run optimal trend inflation is higher in Rotemberg than in Calvo. In both models, a two percent long run inflation target is attainable for a fairly high degree of inflation indexation.

Key words:
Inflation Target, Partial Inflation Indexation, Endogenous Growth, Calvo Pricing, Rotemberg Pricing.

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1. Introduction

In major industrial nations, Central Banks set a long run inflation target around 2%. The well known Friedman rule mandates a negative inflation rate for the nominal interest rate to be zero. Schmitt-Grohe and Uribe (2011) and Woodford (2003) depart from this Friedman rule and argue that the optimal inflation rate is zero in reasonable economic environments. This gives rise to an anomaly because in reality the long run inflation target (trend inflation) is positive and around 2% for major industrial and emerging market economies.

This paper aims to revisit this issue in a New Keynesian endogenous growth setting. The focus of this paper is purely on long run growth and welfare implications of trend inflation and that is why we abstract from any aggregate shocks to the economy. Even though there is a rapidly growing literature on trend inflation[1], little effort is directed to understand the long run growth and welfare effects of a positive inflation targeting in the presence of nominal and real frictions in the economy. In our model, the link between trend inflation and growth is inflation inertia in the form of imperfect inflation indexation. If inflation is not fully indexed, a higher trend inflation redistributes income between monopolistic profit and rental income earners. Through this income redistribution channel, trend inflation impacts long run growth. The quantitative effect of trend inflation on growth depends crucially on the nature of price setting which is the central theme of this study.

As in Ascari and Rossi (2012), we analyze two types of price setting behaviour, namely (i) Calvo (1983) where firms randomly reset prices, and (ii) Rotemberg (1982) where all firms continuously set prices subject to some price adjustment costs. Growth effects of a trend inflation are very different in these two price setting environments. As in King and Wolman (1996), a higher trend inflation has an ambiguous effect on the average price-marginal cost markup of monopolistic intermediate goods firms in a Calvo price setting world. This happens due to the tension between *price adjustment* and *marginal markup effects*. In the Calvo model, this ambiguity trans-

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[1]See Ascari and Sbordone (2014) for a comprehensive survey.

lates into a hump shaped relation between long run growth and trend inflation. On the other hand, in a Rotemberg price setting environment, higher trend inflation unambiguously erodes the average markup of the intermediate goods firms along the same transmission channel described in Ascari and Rossi (2012) and promotes long run growth via boosting the rental price of capital.[2]

The effect of trend inflation on the steady state welfare depends not only how the trend inflation impacts long run growth but also on how the trend inflation influences the price distortion. In the Calvo model, the price distortion arises due to the dispersion of prices among sticky and flexible price firms. In Rotemberg model, the price distortion arises solely due to the price adjustment costs. In both models, price distortion by causing productive inefficiency and loss of resources engenders adverse wealth effect on the representative household and depresses steady state welfare. After factoring the growth and price distortionary effects, our calibrated growth model still yields a positive welfare maximizing inflation in both models. As in Lombardo and Vestin (2008) the inefficiency due to nominal rigidity is higher in the Calvo model than in Rotemberg. The relative output loss in Calvo model progressively rises when the trend inflation is higher. This makes the growth enhancing effect of trend inflation in the Calvo model weaker than in the Rotemberg model. The immediate implication is that the optimal long run inflation is higher in the Rotemberg model than in the Calvo model. Both models yield a positive optimal long run annual inflation rate. A sensitivity analysis of welfare maximizing inflation shows that the optimal inflation is very sensitive to the degree of inflation indexation. Higher inflation indexation raises the optimal inflation rate in both models because more indexation of inflation dampens the price distortionary effects.

In our growth model, the optimality of positive trend inflation rests on a comparison across inefficient steady states. Two types of inefficiencies are present in the steady state of our model economy. The first inefficiency is due to market im-

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[2]Ascari and Rossi (2012) provide useful insights about the differential effects of Calvo and Rotemberg models of price fixing but they do not address the issue of welfare maximizing trend inflation in an endogenous growth setting as we do.

perfection while the second inefficiency results from nominal rigidity. A zero trend inflation eliminates the second inefficiency but could not overcome the first inefficiency. We demonstrate that in such a second best environment, a small amount of trend inflation could be welfare improving.

The rest of the paper is organized as follows. In the following section we review the relevant literature on trend inflation. In section 3, we lay out the basic setup and compare the balanced growth and long run welfare properties of Calvo and Rotemberg models. Section 4 reports the calibration. Section 5 concludes.

2. Literature Review

There is a substantial literature that rationalizes the benefit of a positive trend inflation. Svensson (1997) and Mishkin (2004) argue that a higher trend inflation could reduce inflation volatility and the impact of shocks. Blanchard et al. (2010) and Williams (2009) argue that in the presence of a zero lower bound (ZLB) for the nominal interest rate, a higher long run inflation target gives the Central Bank greater latitude to lower the nominal interest rate. Billi (2011) develops a small open economy new Keynesian model with ZLB for the nominal interest rate and argues that the long run inflation target is very low if the government commits in advance to a future policy plan. On the other hand, if the government follows discretion, the long run inflation target could be inordinately high. Ascari and Sbordone (2014) undertake a comprehensive analysis of the adverse effect of a higher trend inflation on the stability of the aggregate economy. A higher trend inflation shrinks the region of determinacy of the model. In view of this result, they caution about the pitfall of a positive inflation target to mitigate the ZLB problem.

Although this literature provides useful insights about the rationale for a positive trend inflation, it does not factor into account the long run growth consequences of a positive inflation target because of not including capital stock as a reproducible input. Ascari and Sbordone (2014) note that little work has been done on the effects of trend inflation on the aggregate economy with capital. Our novelty is to understand the welfare consequence of trend inflation via the growth channel which necessitates the use of an endogenous growth model. Since the thrust of this paper is on the effect of

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trend inflation on long run growth and welfare, unlike Ascari and Sbordone (2014), we abstract from the effect of trend inflation on the volatility and stability of the economy and only focus on the balanced growth path using a deterministic model of endogenous growth with a simple "AK" technology as in Rebelo (1991). Our model connects to a growing literature that highlights the difference between Calvo and Rotemberg price settings. Ascari and Rossi (2012) focus on the long run new Keynesian Phillips curve. Damjanovic and Nolan (2010), and Leith and Liu (2014) focus on optimal inflation. However, these models do not look into growth effects of monetary policy.

There is a sparse literature exploring the growth and welfare effects of inflation in a new Keynesian endogenous growth setting. Amano, Carter and Moran (2012) have a rich endogenous growth model with nominal price and wage rigidity and find that the negative growth and welfare effects of inflation primarily arise from the distortionary effects of inflation on labour supply. Their model basically highlights the welfare loss of inflation and thus unable to rationalize the positive long run inflation target in a growing economy. Arato (2009) develops an endogenous growth model with endogenous contract duration and explores the growth effect of inflation but does not specifically address the issue of welfare maximizing inflation target. On the other hand, the novelty of our paper is that we use a simple New Keynesian endogenous growth model and after factoring into account the positive and negative effects of trend inflation on steady state welfare, we find that the welfare maximizing trend inflation is still positive and for a plausible price setting environment, it comes close to what we observe in the real world.

3. Basic Setup

We lay out a simple New-Keynesian model with three players, firms, households and the Central Bank. There is a continuum of intermediate goods firms in the economy in the unit interval. Each variety (j) of such goods is produced with a linear technology as follows:

$$x_{jt} = Ak_{jt} \tag{1}$$

where x_{jt} is the amount produced of such good, k_{jt} the capital used in the production and A is a productivity parameter. The linear technology (AK type as in Rebelo, 1991) is the vehicle of endogenous growth. Each variety is produced by a firm with a patent right which disallows the entry of new firms to replicate this variety. Final goods firms transform these intermediate goods into the production of final goods (y_t) using the CES aggregator:

$$y_t = \left[\int_0^1 x_{jt}^{(\sigma-1)/\sigma} dj \right]^{\frac{\sigma}{(\sigma-1)}}. \tag{2}$$

where σ is the elasticity of substitution between intermediate goods.

The household owns the capital and all the firms. It rents capital to intermediate goods firms for production. There is no aggregate risk in this environment. The representative household's maximization problem is given by:

$$Max \quad \sum_{t=0}^{\infty} \beta^t u(c_t)$$

s.t.

$$P_t c_t + P_t(k_{t+1} - (1-\delta)k_t) + B_{t+1} \tag{3}$$
$$= (1+i_t)B_t + R_t k_t + D_t$$

and the usual solvency condition, $\lim_{T \longrightarrow \infty} E_t \frac{\beta^T u'(c_{t+T})}{u'(c_t)} \frac{B_{t+T}}{P_{t+T}} \geq 0$ for all t. Notations are: t=time, c_t=per capita real consumption, P_t =nominal price, k_t=average capital stock, B_t=stock of nominal one period discount bonds in zero net supply, i_t =nominal interest rate, R_t = nominal rental price of capital, D_t =nominal profit from the intermediate goods firms, δ is the fractional rate of depreciation and β is the subjective discount factor in the unit interval.[3]

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[3]The long run property of the model is invariant to external habit formation and investment adjustment cost, which we ignore for simplicity.

The Euler equations of the household are given by:

$$k_{t+1}: \ u'(c_t) = \beta u'(c_{t+1})(1 + r_{t+1} - \delta) \tag{4}$$

$$B_{t+1}: \ u'(c_t) = \beta u'(c_{t+1})(1 + i_{t+1})(P_t/P_{t+1}) \tag{5}$$

where r_t is the real rental price of capital equal to R_t/P_t.

Using (4) and (5) one obtains:

$$(1 + i_{t+1}) = (1 + r_{t+1} - \delta)(P_{t+1}/P_t) \tag{6}$$

The Central Bank (CB) sets a long run target Π (i.e., trend inflation) that satisfies the arbitrage condition (6). This means that along a balanced growth path the CB sets an interest rate as follows:

$$1 + i = (1 + r - \delta)\Pi$$

3.1. Balanced Growth

In equilibrium, all markets clear which means, $c_t + k_{t+1} - (1 - \delta)k_t = y_t$ and $B_t = 0$. Using the household's Euler equation (4) and assuming a logarithmic utility function, $u(c_t) = \ln c_t$, one obtains the balanced growth rate, G:

$$G = \beta(1 + r - \delta) \tag{7}$$

To accommodate this balanced growth(G) and the inflation target (Π), the CB lets the money supply grow at a rate ΠG.

3.2. Calvo Model

We now turn to the price setting scenario of Calvo (1983). In this model, all intermediate goods firms facing the same technology are ex ante identical. Each period a firm receives a random "price change" signal with a probability $1 - \theta$. In the spirit of Yun (1996), if the intermediate goods firm does not receive a price signal, its price is increased at the steady state rate of inflation (Π) subject to an inflation

indexation parameterized by $\gamma \in (0,1)$. Lower γ means less indexation or greater inflation inertia. The partial inflation indexation formulation is borrowed from Smets and Wouters (2003).

The cost minimization from the final goods sector yields the conditional input demand functions:

$$x_{jt} = \left(\frac{P_{jt}}{P_t}\right)^{-\sigma} y_t \tag{8}$$

where $P_{jt} = \Pi^\gamma P_{jt-1}$ if $j \in (0, \theta)$ and $P_{jt} = P_t^*$ otherwise. P_t is the general price level at date t.

Firms solve the optimal price setting problem:

$$Max \sum_{k=0}^{\infty} \theta^k M_{t,t+k} (\Pi^{\gamma k} P_t^* x_{t+k|t} - TC_{t+k|t}(x_{t+k|t}))$$

subject to the demand functions,

$$x_{t+k|t} = \left(\frac{\Pi^{\gamma k} P_t^*}{P_{t+k}}\right)^{-\sigma} y_{t+k}$$

where $M_{t,t+k}$ is the firm's nominal discount factor and $TC_{t+k|t}$ is the price setter's forecast of the nominal total cost at time $t + k$ conditional on the information at date t.

The optimal price (P_t^*) is nonstationary and thus it is normalized by the general price level P_t. It is straightforward to verify that:

$$\frac{P_t^*}{P_t} = \left(\frac{\sigma}{\sigma - 1}\right) \frac{\sum_{k=0}^{\infty} (\theta \Pi^{-\sigma\gamma})^k M_{t,t+k} \Pi_{t,t+k}^{\sigma+1} mc_{t,t+k} \left(\frac{y_{t+k}}{y_t}\right)}{\sum_{k=0}^{\infty} (\theta \Pi^{\gamma(1-\sigma)})^k M_{t,t+k} \Pi_{t,t+k}^{\sigma} \left(\frac{y_{t+k}}{y_t}\right)} \tag{9}$$

where $mc_{t,t+k}$ is the k-period ahead forecast of the real marginal cost. Given the linear production function (1), $mc_{t,t+k} = r_{t,t+k}/A$ where $r_{t,t+k}$ is the k-period ahead forecast of the real rental price of capital. where $mc_{t,t+k}$ is the k-period ahead forecast of the real marginal cost. Given the linear production function (1), $mc_{t,t+k} = r_{t,t+k}/A$

where $r_{t,t+k}$ is the k-period ahead forecast of the real rental price of capital.

The law of motion of the general price level is given by:

$$P_t = \left[\theta(\Pi^\gamma P_{t-1})^{1-\sigma} + (1-\theta)P_t^{*1-\sigma}\right]^{\frac{1}{1-\sigma}} \tag{10}$$

Based on the same principle as in King and Wolman (1996), one gets the following expression for the average markup along the balanced growth path (BGP).[4]

$$\frac{P_t}{MC_t} = \left(\frac{P_t^*}{MC_t}\right)\left(\frac{P_t}{P_t^*}\right)$$

$$= \mu_n\left(\frac{P_t}{P_t^*}\right) \tag{11}$$

where MC_t is the nominal marginal cost and using (9) μ_n is the marginal markup defined as

$$\mu_n = \frac{\sigma}{\sigma-1}\left[\frac{1-\theta\beta\Pi^{(1-\gamma)(\sigma-1)}}{1-\theta\beta\Pi^{(1-\gamma)\sigma}}\right] \tag{12}$$

and the price adjustment gap, P_t/P_t^* is given by (based on (10))

$$\frac{P_t}{P_t^*} = \left[\frac{1-\theta\Pi^{(1-\gamma)(\sigma-1)}}{1-\theta}\right]^{1/(\sigma-1)} \tag{13}$$

Higher trend inflation erodes the markup in a world with imperfect inflation indexation as seen from the price adjustment gap term (13). To combat this, price setting firm raises the marginal markup.[5] As in King and Wolman (1996), the effect of a higher trend inflation on the average markup is thus ambiguous due to the conflicting effects on the marginal markup and the price adjustment gap.

The ambiguous effect of trend inflation on average markup could potentially give rise to a hump shaped relationship between the long run growth and trend inflation.

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[4]The appendix provides a derivation of (11). For the steady state price ratio to exist one needs the convergence condition that $\Pi < \theta^{\frac{1}{(\gamma-1)\sigma}}$ which is 11.39% for the baseline parameter values.

[5]The appendix shows that $\partial\mu_n/\partial\Pi > 0$ as long as $\Pi > 1$.

This happens because the balanced growth (7) is driven by the real rental price of capital (r) which equals $A.mc$. Since the real marginal cost, mc is the reciprocal of the average markup P_t/MC_t, a higher trend inflation has an ambiguous effect on the balanced growth rate.

3.3. Rotemberg Model

In Rotemberg (1982) all firms continuously adjust prices but all of them are subject to a quadratic price adjustment cost measured in terms of final goods. We follow Ascari and Rossi (2012) and Ireland (2007) in specifying the price adjustment cost function subject to imperfect inflation indexation as follows:

$$\frac{\varphi}{2}\left(\frac{P_{jt}}{\Pi^\gamma P_{jt-1}} - 1\right)^2 y_t \tag{14}$$

where $\varphi > 0$ is the degree of nominal rigidity, Π^γ represents the indexation of the last period price level based on the trend inflation, Π, and γ is the degree of price indexation as before.

The optimal price fixing problem of each intermediate goods firm is, therefore,

$$Max \sum_{k=0}^{\infty} M_{t,t+k}\left[P_{jt+k}x_{j+k} - MC_{t+k}x_{jt+k} - \frac{\varphi}{2}\left(\frac{P_{jt}}{\Pi^\gamma P_{jt-1}} - 1\right)^2 P_{t+k}y_{t+k}\right] \tag{15}$$

subject to the same sequence of demand functions as in (8). Since all firms face the same technology and the same price adjustment costs, there is no heterogeneity in price fixing behaviour as in Calvo. Thus in a symmetric equilibrium $P_{jt} = P_t$ for all j. Unlike Calvo (1983), there is no difference between average and marginal markup because all firms charge the same price. Along the BGP, average markup is given by (the proof is relegated to the appendix):

$$\frac{P_t}{MC_t} = \left[\frac{\varphi(1-\beta)\Pi^{1-\gamma}(\Pi^{1-\gamma}-1)}{\sigma} + \frac{\sigma-1}{\sigma}\right]^{-1} \tag{16}$$

Higher trend inflation (Π) lowers the markup due to imperfect inflation indexation. Since the steady state rental price (r) is $A.mc$, this means that a higher trend infla-

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tion unambiguously raises the balanced growth rate in the Rotemberg price setting scenario.[6]

3.4. Long run Welfare in Calvo and Rotemberg Models

For any arbitrary balanced growth (G), the steady state welfare (W) function is given by:

$$W = \frac{\ln c_0}{1 - \beta} + \frac{\beta \ln G}{(1 - \beta)^2} \tag{17}$$

where c_0 is the initial consumption.

The initial consumption (c_0) and the balanced growth rate (G) differ between Calvo and Rotemberg price setting regimes. Hereafter we distinguish between these two regimes with superscripts C and R respectively. In the Calvo scenario, the balanced growth rate is given by[7]:

$$G^C = \beta \left[A \left\{ \mu_n \frac{P_t}{P_t^*} \right\}^{-1} + 1 - \delta \right] \tag{18}$$

and the initial consumption (the proof is relegated to the appendix) is given by

$$c_0^C = [(A \bar{s}^{-1} + 1 - \delta) - G^C] \tag{19}$$

where $\bar{s}$ is the steady state price dispersion given by,[8]

$$\bar{s} = \frac{(1 - \theta \Pi^{(1-\gamma)(\sigma-1)})^{\sigma/(\sigma-1)}}{(1 - \theta \Pi^{\sigma(1-\gamma)})(1 - \theta)^{1/(\sigma-1)}} \tag{20}$$

The steady state welfare depends on the long run inflation rate (Π) through two channels: (i) the balanced growth (G) and (ii) relative price dispersion channel $(\bar{s})$. The trend inflation has ambiguous effect on growth due to conflict between

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[6]Note that growth cannot indefinitely rise because a lower markup depresses the profits of the intermediate goods firms which adversely affects household's consumption.

[7]The details of the derivation of growth and welfare functions are relegated to the appendix.

[8]See the appendix for the derivation of the steady state price dispersion.

marginal markup and price adjustment effects. On the other hand, a higher trend inflation elevates the steady state price dispersion along the same channel described in Ascari and Sbordone (2014).[9] Although this depresses the initial consumption via the second term of (19), the ambiguous inflation effects on growth makes the sign of the comparative statics $\partial c_0^C / \partial \Pi$ ambiguous. The overall effect of trend inflation on long run welfare is thus ambiguous in the Calvo model.

For the Rotemberg model, the balanced growth rate and initial consumption are given by:

$$G^R = \beta \left[A \left\{ \frac{\varphi(1-\beta)\Pi^{1-\gamma}(\Pi^{1-\gamma}-1)}{\sigma} + \frac{\sigma-1}{\sigma} \right\} + 1 - \delta \right] \tag{21}$$

and the initial consumption is given by (the proof is relegated to the appendix):

$$c_0^R = [(A(1-\tau) + 1 - \delta) - G^R]k_0 \tag{22}$$

where

$$\tau = \frac{\varphi}{2} \left(\Pi^{1-\gamma} - 1 \right)^2 \tag{23}$$

Since all firms are homogeneous, there is no price dispersion in the Rotemberg model. Instead of price dispersion, the price adjustment cost acts as an implicit tax on TFP. Higher trend inflation depresses household's initial consumption through two channels. First it raises the price adjustment cost and acts as a tax on output. Second, it unambiguously raises the steady state rental price of capital (meaning a higher G^R). On the other hand, the household's initial consumption c_0^R falls. The effect on the steady state welfare is again ambiguous.

4. Calibration

To assess the relationship between W and Π quantitatively, we fix $\beta = 0.99$ and $\delta = 0.025$ at the conventional levels. The demand elasticity parameter σ is fixed at

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[9]See the appendix for a proof.

6.00 as in Kollmann (2002). The productivity parameter A is fixed to target the long run per capita quarterly real GDP growth rate at 0.49% which means an annualized growth rate of 1.97% for the sample period 1947-2014.[10]There is considerable disagreement in the literature about the range of values for the price stickiness parameter, θ. Kollmann (2002) uses 0.75 as the baseline value while Smets and Wouters (2003) estimate a higher value of θ around 0.91. These values basically mean that the average duration of prices to remain sticky is 4 to 10 quarters. We take an average of these extreme values and set θ equal to 0.85 as a baseline.

A similar ambiguity arises about the size of the inflation indexation parameter γ. For Euro regions, Smets and Wouters (2003) estimate γ around 0.52. Using GMM approach Sahuc (2004) comes up with an estimate of γ around 0.41 for Euro regions and 0.64 for the US. As a baseline we start off with a conservative estimate, $\gamma = 0.52$ which is close to Smets and Wouters (2003) estimate or an average of the Euro and US estimates of Sahuc (2004). We then carry out a sensitivity analysis to check how the optimal long run inflation depends on the size of γ.

It is difficult to find an estimate of the price adjustment cost parameter, φ, that is consistent with our growth model. Keen and Wang (2005) calibrate this by matching the slopes of the New Keynesian Phillips curves from Calvo and Rotemberg models. In our context, balanced growth rate is a crucial link between Calvo and Rotemberg models. In a similar vein, we calibrate φ by matching the balanced growth rates, G^C and G^R which yields an analytical expression for φ as follows:

$$\varphi = \left[\left\{\frac{\sigma}{\sigma-1}\mu_n\frac{P_t}{P_t^*}\right\} - \frac{\sigma-1}{\sigma}\right]\left[\frac{\sigma}{(1-\beta)\Pi^{1-\gamma}(\Pi^{1-\gamma}-1)}\right] \tag{24}$$

The price adjustment cost parameter depends nonlinearly on the trend inflation Π. As a baseline we evaluate φ at a zero inflation level which yields[11]:

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[10]Data for annual per capita real GDP in chained 2009 US dollars came from the Bureau of Economic Analysis.

[11]The expression for φ in eq (psi1) comes from an application of L'Hosptial's rule to (24).

$$\varphi = \left(\frac{\sigma-1}{1-\beta}\right)\left(\frac{\theta}{1-\theta} - \frac{\theta\beta}{1-\theta\beta}\right) \tag{25}$$

Note that φ is increasing in θ and not surprisingly at zero inflation steady state, inflation indexation parameter, γ plays no role in determining φ. For our calibrated values of σ, β and θ, the price adjustment cost parameter, φ equals 178.76. This value is close to Keen and Wang (2005) for a price markup in the range 10 to 20% and Calvo nominal rigidity parameter θ around 0.8.[12]

Figure 1 plots the growth effect of trend inflation for the Calvo model where both growth and inflation rates are annualized. The hump shaped relationship arises which bears out the intuition described earlier about the conflict between marginal markup effect and price adjustment effect caused by a higher trend inflation. The positive price adjustment effect dominates first and then the marginal markup effect picks up. The growth maximizing inflation is around 1.49%.
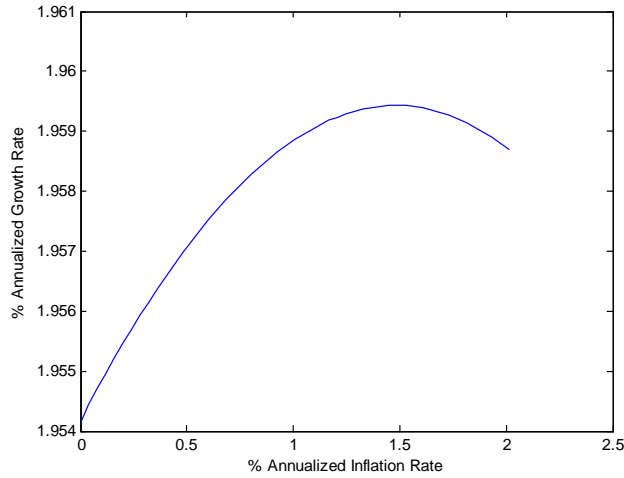

Figure 1: Growth Effects of Inflation in the Calvo Model

Figure 2 plots the welfare effect of annualized trend inflation in the Calvo model. A welfare maximizing inflation results around 0.52% after balancing the negative

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[12]The balanced growth rate in our calibrated Rotemberg model is not very sensitive to the value of φ.

price distortion effects and the hump-shaped effects of trend inflation on growth. Not surprisingly the welfare maximizing inflation is lower than the growth maximizing inflation because of an additional negative price distortionary effect on the steady state welfare.
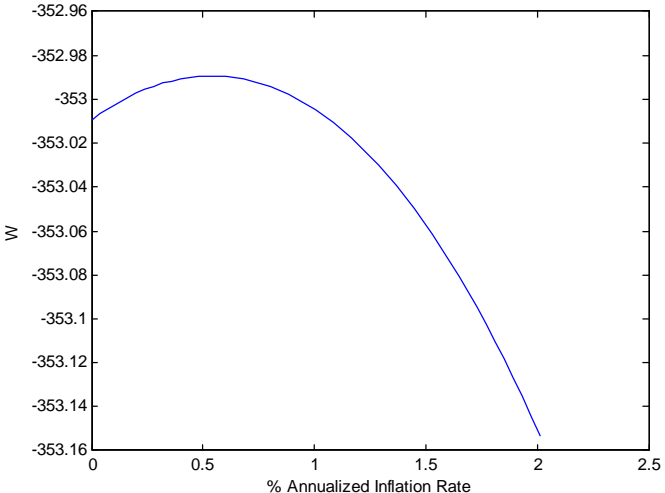

Figure 2: Welfare maximizing inflation in the Calvo model

Figure 3 plots the effect of trend inflation on the balanced growth in the Rotemberg model. Since the average price markup unambiguously decreases due to higher inflation, the steady state rental price rises which promotes growth unambiguously.
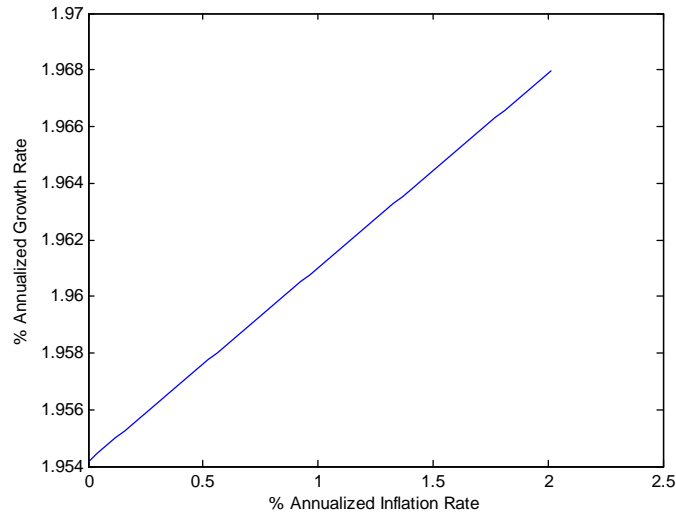
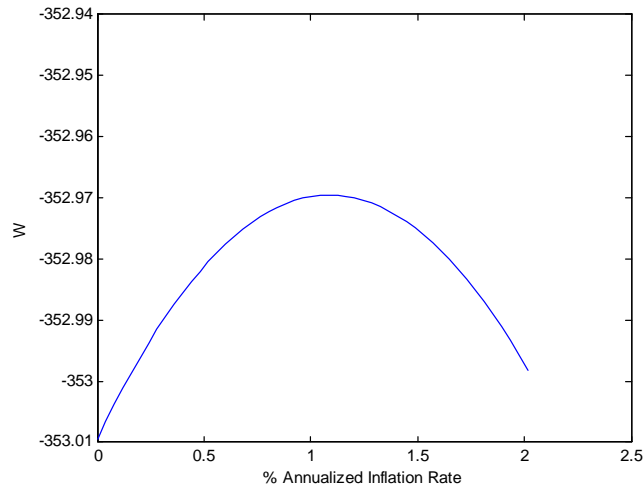Figure 3: Growth Effects of Inflation in the Rotemberg Model

Figure 4: Welfare effects of trend inflation in the Rotemberg model

Figure 4 plots the effect of trend inflation on the steady state welfare in the Rotemberg model. A hump shaped relationship again emerges which picks at 1.08%. The welfare maximizing inflation in Rotemberg model is higher than the Calvo model. In both models, the growth at the welfare maximizing inflation is 1.96% on par with the annual growth rate in the US.

To understand the reasons for the higher optimal long run inflation in Rotemberg model compared to Calvo, we compute the inefficiency in both models due to nominal rigidity. In the Calvo model the efficiency loss arises due to the price dispersion, $\bar{s}$ as shown in (20). In the appendix, we demonstrate that the steady state implicit tax on TFP due to price dispersion is given by $(\bar{s} - 1)/\bar{s}$. On the other hand, in the Rotemberg model the efficiency loss arises due to the price adjustment cost which results in an implicit steady state tax on TFP equal to $\frac{\varphi}{2} \left(\Pi^{1-\gamma} - 1 \right)^2$ as seen from (22) and (23). Figure 5 plots the ratio of these two implicit taxes in Calvo and Rotemberg for a range of annualized trend inflation rates. Higher inflation unambiguously raises the inefficiency in the Calvo model compared to Rotemberg. This progressive relative output loss in the Calvo model vis-a-vis Rotemberg is at the very foundation of a higher optimal long run inflation in the Rotemberg model.
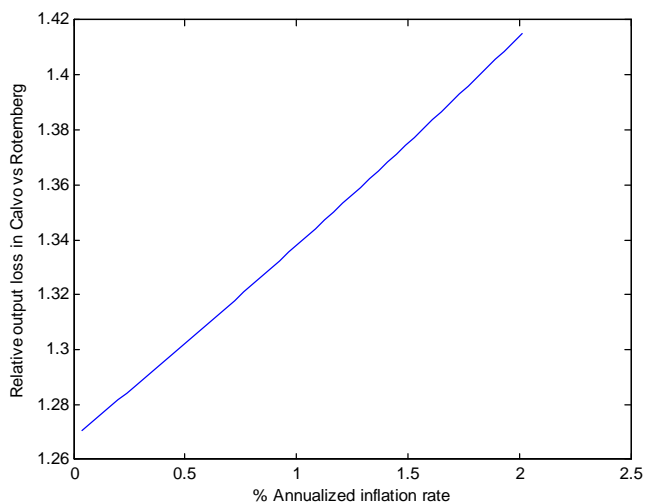

Figure 5: Relative output loss in Calvo vs Rotemberg due to nominal rigidity

Table 1 demonstrates the sensitivity of the welfare maximizing inflation with respect to θ and γ for these two models. The welfare maximizing inflation is sensitive to the degree of inflation indexation. A higher indexation raises the optimal inflation because it dampens the negative distortionary effect of trend inflation on welfare in both Calvo and Rotemberg models (see eqs (20) and (23)). For $\gamma = 0.74$, the Calvo

model reproduces an optimal trend inflation around 2%. For the Rotemberg model, the 2% target inflation is obtained for a value of γ around 0.90.

A 2% long run inflation target is, therefore, obtained for a value of the inflation indexation parameter γ higher than standard estimates in the literature. Is such a high value of γ plausible? Note that in the context of our long run growth model we do not distinguish between short run and long run indexation rules for inflation. One can interpret γ as a weighted average of short run and long run indexation parameters. To see it clearly, in a similar spirit as in Ascari and Rossi (2012) let the indexation rule (call it Θ_t) be based on the past and long run inflation as follows.

$$\Theta_t = \left(\Pi_{t-1}^{\omega_1}\right)^{\mu} \left(\Pi^{\omega_2}\right)^{1-\mu} \tag{26}$$

where the past period inflation Π_{t-1} is indexed by a fraction ω_1, trend inflation Π is indexed by the fraction ω_2; μ and $(1-\mu)$ are the relative weights assigned to $\Pi_{t-1}^{\omega_1}$ and Π^{ω_2} respectively. The indexation parameters ω_1 and ω_2 are the respective degrees of short run and long run inflation indexation. Ascari and Rossi (2012) assume that $\omega_1 = \omega_2$ which presupposes that the agent attaches same indexation to both past and trend inflation. We can make the indexation rule (26) more general by allowing the short run indexation (ω_1) to differ from the long run indexation (ω_2).[13] In the context of our long run model (where $\Pi = \Pi_{t-1} \ \forall \ t$), the indexation formula (26) then reduces to

$$\Theta = \Pi^{\gamma} \tag{27}$$

where $\gamma = \mu\omega_1 + (1-\mu)\omega_2$. Viewed from this perspective, a high value of the inflation indexation parameter, γ is not implausible if one interprets it as a weighted average of short run and long run indexation rules.

<Table 1 comes here>

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[13]Such an indexation formulation encompasses other formulations in the extant literature as special cases. For example, if $\mu = 1$, it reduces to Smets and Wouters (2003). If $\mu = 0$ and $\omega_2 = 1$, we get Yun (1996) full indexation

4.1. Why is positive trend inflation welfare improving?

In both Calvo and Rotemberg models price distortions either in the form of price dispersion or price adjustment cost give rise to misallocation of resources. The question arises: why does not the Central Bank eliminate this distortion right at the outset by setting the trend inflation equal to zero? Setting trend inflation to zero would eliminate the nominal friction and lead the economy to a flexible price steady state which is still subject to real friction arising from market imperfection. Such a steady state is, therefore, an inefficient steady state due to the existence of market imperfection. If one compares a zero inflation inefficient steady state with an inefficient steady state with a small dose of trend inflation, the latter could be welfare improving. To see this more clearly first note that the real rental price in a flexible price inefficient steady state is given by $A(\sigma - 1)/\sigma$. The reciprocal of the flexible price markup imposes a tax on the TFP. The immediate consequence is that the balanced growth is given by:

$$G = \beta(A(\sigma - 1)/\sigma + 1 - \delta) \tag{28}$$

As seen from Figures 1 and 3, a positive trend inflation up to a threshold could improve the long run growth rate in both Calvo and Rotemberg models compared to a flexible price inefficient balanced growth rate as in (28).

The welfare comparison is less straightforward. To compare the steady state welfare between zero and positive inflation inefficient steady states, we need to first derive the steady state welfare function for the flexible price steady state. Given our logutility function and the linear production function, the optimal consumption and investment rules in a flexible price steady state are given by the Solow saving rules as follows:

$$k_{t+1} = \beta(A(\sigma - 1)/\sigma + 1 - \delta)k_t \tag{29}$$

and

$$c_t = \left[A(1 - \beta\frac{\sigma - 1}{\sigma}) + (1 - \delta)(1 - \beta) \right] k_t \tag{30}$$

The appendix outlines the derivations of (29) and (30). Plugging the consumption decision rule (30) and the balanced growth (28) into the welfare function (17) and normalizing the initial capital stock (k_0) to unity one gets

$$W^{flex} = \frac{\ln\left[A(1 - \beta^{\frac{(\sigma-1)}{\sigma}}) + (1 - \delta)(1 - \beta)\right]}{1 - \beta} + \frac{\beta \ln G}{(1 - \beta)^2}$$

where W^{flex} represents the steady state welfare in an inefficient flexible price economy. For the baseline values of the parameters, we find that $W^{flex} = -353.0095$. In the Calvo model, as seen from Figure 2 the steady state welfare at the optimal inflation is -352.9894 and for the Rotemberg model, the same is -352.9696 as seen from Figure 4. Thus a flexible price zero inflation steady state welfare is lower than a positive inflation steady state welfare for our calibrated economy. The policy authority can do better in inflating the economy a bit compared to a zero inflation flexible price steady state.

5. Conclusion

The inflation targets of the major industrial nations are consistently above zero which goes against the conventional wisdom of zero or negative inflation based on welfare theoretic considerations. We set up a new-Keynesian endogenous growth model to address this apparent anomaly. Due to inflation inertia in the form of partial inflation indexation, a higher long run inflation gives rise to opposing effects on welfare via a positive growth effect up to a threshold inflation and a negative price distortionary effects. An optimal inflation rate exists which maximizes the long run aggregate welfare. For plausible parameter values we find that this optimal inflation rate is positive. The level of this inflation depends on the nature of price setting in the model. The welfare maximizing inflation is consistently higher in the Rotemberg model compared to Calvo due to greater inefficiency in the latter caused by price dispersion. A future extension of this paper would be to examine the short run implications of inflation targets in the presence of aggregate shocks.

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A. Appendix

Derivation of equation (11)

Assuming a logarithmic utility function of the form $u\left(c_{t+k}\right) = \ln\left(c_{t+k}\right)$, the nominal discount factor (9) can be written as:

$$M_{t,t+k} = \beta^k \left(\frac{c_t}{c_{t+k}}\right)\left(\frac{P_t}{P_{t+k}}\right) \tag{A.1}$$

which can be written along the BGP as:

$$M = \left(\frac{\beta}{\Pi G}\right)^k \tag{A.2}$$

Along the BGP, the numerator of (9), $mc\sum_{k=0}^{\infty}(\theta\beta\Pi^{\sigma(1-\gamma)})^k = \frac{mc}{1-\theta\beta\Pi^{\sigma(1-\gamma)}}$ and the denominator is $\sum_{k=0}^{\infty}\left(\theta\beta\Pi^{(\sigma-1)(1-\gamma)}\right)^k = \frac{1}{1-\theta\beta\Pi^{(\sigma-1)(1-\gamma)}}$.

Thus along the BGP, the optimal price setting equation reduces to:

$$\frac{P_t^*}{P_t} = \left(\frac{\sigma}{\sigma-1}\right)\left(\frac{1-\theta\beta\Pi^{(\sigma-1)(1-\gamma)}}{1-\theta\beta\Pi^{\sigma(1-\gamma)}}\right)mc$$

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which implies

$$\frac{P_t}{MC_t} = \mu_n \frac{P_t}{P_t^*}$$

same as (11).

Derivation of $\frac{\partial \ln \mu_n}{\partial \Pi} > 0$

We have

$$\mu_n = \frac{\sigma}{\sigma - 1} \left[\frac{1 - \theta\beta\Pi^{(1-\gamma)(\sigma-1)}}{1 - \theta\beta\Pi^{(1-\gamma)\sigma}} \right]$$

Therefore,

$$\ln \mu_n = \ln \left(\frac{\sigma}{\sigma - 1} \right) + \ln \left(1 - \theta\beta\Pi^{(1-\gamma)(\sigma-1)} \right) - \ln \left(1 - \theta\beta\Pi^{(1-\gamma)\sigma} \right)$$

Next note that for $\Pi > 1$,

$$\frac{\partial \ln \mu_n}{\partial \Pi} = \frac{\theta\beta(1-\gamma)\sigma\Pi^{(1-\gamma)\sigma-1}}{1 - \theta\beta\Pi^{(1-\gamma)\sigma}} - \frac{\theta\beta(1-\gamma)(\sigma-1)\Pi^{(1-\gamma)(\sigma-1)-1}}{1 - \theta\beta\Pi^{(1-\gamma)(\sigma-1)}} > 1$$

Derivation of (16)

The first order condition for this price setting problem (15) of the jth intermediate goods firm yields:

$$(1 - \sigma) \left(\frac{P_{jt}}{P_t} \right)^{-\sigma} y_t + \frac{r_t}{A}\sigma \left(\frac{P_{jt}}{P_t} \right)^{-\sigma-1} y_t - \varphi P_t \left(\frac{P_{jt}}{\Pi^\gamma P_{jt-1}} - 1 \right) \frac{y_t}{\Pi^\gamma P_{jt-1}} +$$

$$\beta \frac{u'(c_{t+1})}{u'(c_t)} \varphi P_t \left(\Pi^{-\gamma} \frac{P_{jt+1}}{P_{jt}} - 1 \right) \Pi^{-\gamma} P_{jt+1} P_{jt}^{-2} y_{t+1} = 0 \qquad \text{(A.3)}$$

Since all firms are homogeneous, they charge the same price in a symmetric equilibrium which means $P_{jt} = P_t$. Eq (A.3) thus reduces to

$$(1-\sigma)y_t + \frac{r_t}{A}\sigma y_t - \frac{\varphi\Pi_t}{\Pi^\gamma}\left(\frac{\Pi_t}{\Pi^\gamma} - 1 \right) y_t + \beta\frac{u'(c_{t+1})}{u'(c_t)}\varphi\Pi^{-\gamma}\Pi_t \left(\Pi^{-\gamma}\Pi_t - 1 \right) y_{t+1} = 0 \quad \text{(A.4)}$$

Along the BGP, $\Pi_t = \Pi$, $r_t = r$ and $\frac{y_{t+1}}{y_t} = \frac{c_{t+1}}{c_t} = G$. With log utility, eq (A.4)

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reduces to

$$\frac{r}{A} = \left[\frac{\varphi(1-\beta)\Pi^{1-\gamma}(\Pi^{1-\gamma}-1)}{\sigma} + \frac{\sigma-1}{\sigma}\right]$$

Since $\frac{r}{A} = mc_t = \frac{MC_t}{P_t}$, from the above equation the steady state average mark up can be expressed as

$$\frac{P_t}{MC_t} = \left[\frac{\varphi(1-\beta)\Pi^{1-\gamma}(\Pi^{1-\gamma}-1)}{\sigma} + \frac{\sigma-1}{\sigma}\right]^{-1}$$

which is equation (16).

Derivation of the steady state welfare function for Calvo and Rotemberg models

Along the BGP the steady state welfare can be written as:

$$W = \sum_{t=0}^{\infty} \beta^t \ln c_0 G^t \tag{A.5}$$
$$= \frac{\ln c_0}{1-\beta} + \frac{\beta \ln G}{(1-\beta)^2}$$

The equilibrium resource constraint (3) facing the household is given by:

$$c_t + k_{t+1} - (1-\delta)k_t = r_t k_t + d_t \tag{A.6}$$

Dividing through by k_t and using the balanced growth condition:

$$\frac{c_t}{k_t} + G = (r+1-\delta) + \frac{d_t}{k_t} \tag{A.7}$$

To derive the expression for (19), first aggregate the capital of all firms as:

$$k_t = \int_0^1 k_{jt} d_j$$

Using (1) and (8):

$$k_t = y_t A^{-1} \int_0^1 \left(\frac{P_{jt}}{P_t} \right)^{-\sigma} d_j$$

$$= y_t A^{-1} s_t$$

This means that the final goods production function reduces to:

$$y_t = A s_t^{-1} k_t$$

Next note that the dividend in the Calvo model is given by:

$$d_t = y_t - r_t k_t$$

$$= A s_t^{-1} k_t - r_t k_t$$

$$= (A s_t^{-1} - r_t) k_t$$

which implies

$$\frac{d_t}{k_t} = (A s_t^{-1} - r_t)$$

In the steady state $s_t = \bar{s}$ given by (A.11) which means that

$\frac{c_t}{k_t} + G = (r+1-\delta) + \frac{d_t}{k_t} \Rightarrow \frac{c_t}{k_t} = (r+1-\delta) - G + (A\bar{s}^{-1} - r) = (\frac{1-\beta}{\beta}) G + (A\bar{s}^{-1} - r).$

From (A.7) this implies (given the normalization $k_0 = 1$):

$$c_0^c = [(A\bar{s}^{-1} + 1 - \delta) - G^C] \tag{A.8}$$

In the following section, we derive the expression for $\bar{s}$.

For the Rotemberg model, along the BGP the dividend is given by

$$\frac{d_t}{k_t} = (A(1-\tau) - r)$$

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where τ is given by (23). From (A.7) this implies (given the normalization $k_0 = 1$)

$$c_0^R = [(A(1-\tau) + 1 - \delta) - G^R]$$

Derivation of (20)

We follow Schmitt-Grohe and Uribe (2011). Define the price dispersion as:

$$s_t = \int_0^1 \left(\frac{P_{jt}}{P_t}\right)^{-\sigma} dj \tag{A.9}$$

Using (3) in the text, rewrite (A.9) as a recursion as:

$$s_t = \Pi^{-\gamma\sigma}\Pi_t^\sigma \theta s_{t-1} + (1-\theta)\left(\frac{P_t^*}{P_t}\right)^{-\sigma} \tag{A.10}$$

Plugging $\Pi_t = \Pi$ and (15) into (A.10), we get the following steady state price dispersion (subject to the same convergence condition in footnote 4):

$$\overline{s} = \frac{(1 - \theta\Pi^{(1-\gamma)(\sigma-1)})^{\sigma/(\sigma-1)}}{(1 - \theta\Pi^{\sigma(1-\gamma)})(1-\theta)^{1/(\sigma-1)}} \tag{A.11}$$

Check that

$$\frac{\partial \ln \overline{s}}{\partial \Pi} = \frac{\theta\sigma(1-\gamma)\Pi^{\sigma(1-\gamma)}(1 - \Pi^{-1})}{\Pi(1 - \theta\Pi^{\sigma(1-\gamma)})(1 - \theta\Pi^{(1-\gamma)(\sigma-1)})} > 0 \text{ if } \Pi > 1 \tag{A.12}$$

The denominator is positive given that the convergence condition holds.

Derivation of (29) and (30)

In a flexible price steady state with log utility, the rental Euler equation (4) can be written as:

$$\frac{1}{c_t} = \beta E_t\left[\frac{1 + r - \delta}{c_{t+1}}\right] \tag{A.13}$$

where

$$r = \left(\frac{\sigma - 1}{\sigma}\right) A$$

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We make the following conjectures for the optimal consumption and investment policy rules:

$$c_t = \lambda(A + 1 - \delta)k_t$$

and

$$k_{t+1} = (1 - \lambda)(A + 1 - \delta)k_t$$

Plugging these conjectures in the Euler equation in (A.13), we can solve λ as

$$\lambda = \frac{A(1 - \beta^{\frac{\sigma-1}{\sigma}}) + (1 - \delta)(1 - \beta)}{A + 1 - \delta}$$

which yields (30) and (29).

Table 1: Sensitivity Analysis of the Optimal t Inflation Rate in Calvo and Rotemberg Models (in percent)

	$\theta = 0.75$	$\theta = 0.80$	$\theta = 0.85$	$\theta = 0.90$
$\gamma = 0.40$	Calvo = 0.44	Calvo = 0.44	Calvo = 0.44	Calvo = 0.44
	Rotem= 0.88	Rotem= 0.88	Rotem= 0.88	Rotem= 0.88
$\gamma = 0.50$	Calvo = 0.52	Calvo = 0.52	Calvo = 0.52	Calvo = 0.52
	Rotem = 1.04	Rotem = 1.04	Rotem = 1.04	Rotem = 1.04
$\gamma = 0.60$	Calvo = 0.64	Calvo = 0.64	Calvo = 0.64	Calvo = 0.64
	Rotem = 1.33	Rotem = 1.33	Rotem = 1.33	Rotem = 1.29
$\gamma = 0.70$	Calvo = 0.88	Calvo = 0.88	Calvo = 0.88	Calvo = 0.84
	Rotem = 1.73	Rotem = 1.73	Rotem = 1.73	Rotem = 1.73
$\gamma = 0.74$	Calvo = **2.02**	Calvo = **2.02**	Calvo = **2.02**	Calvo = **2.02**
	Rotem = 1.04	Rotem = 1.04	Rotem = 1.04	Rotem = 1.04
$\gamma = 0.80$	Calvo = 1.33	Calvo = 1.33	Calvo = 1.29	Calvo = 1.29
	Rotem = 2.63	Rotem = 2.63	Rotem = 2.63	Rotem = 2.63
$\gamma = 0.90$	Calvo = **2.02**	Calvo = **2.02**	Calvo = **2.02**	Calvo = **2.02**
	Rotem = 5.34	Rotem = 5.34	Rotem = 5.34	Rotem = 5.34